Problem Type 10.2a: Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter

\[ x = f(t) \quad , \quad y = g(t) \quad ; \quad t = a \quad , \]

where \( f(t), g(t) \) are expressions in \( t \) and \( a \) is some number.

Example Problem 10.2a: Find an equation of the tangent line to the curve at the point corresponding to the given value of the parameter

\[ x = 2t^2 + 1, \quad y = \frac{1}{3}t^3 - t \quad ; \quad t = 3 \quad . \]

Steps

1. Compute \( \frac{dy}{dt} \) and \( \frac{dx}{dt} \), getting expressions in \( t \). Then divide the former by the latter to get \( \frac{dy}{dx} \), a certain expression in \( t \).

You use

\[ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \]

2. Plug-in \( t = a \) to get the slope at the point. Also plug-in \( t = a \) into \( x \) and \( y \) to get the point.

2. When \( t = 3 \), the slope is \( \frac{dy}{dx} = \frac{(t^2 - 1)/4t}{(t^2 - 1)/4t} \), that equals \( (3^2 - 1)/(4 \cdot 3) = \frac{2}{3} \). So \( m = \frac{2}{3} \). To get the point we have \( x(3) = 2 \cdot (3)^2 + 1 = 19 \) and \( y(3) = (1/3) \cdot 3^3 - 3 = 6 \). So the point is \((19,6)\).

3. Use the famous point-slope equation from geometry to find the equation of the line: \( (y - y_0) = m(x - x_0) \).

3. \( y - 6 = (2/3)(x - 19) \), that simplifies to \( y = \frac{2}{3}x - 20/3 \).

Ans.: The equation of the tangent line is \( y = \frac{2}{3}x - \frac{20}{3} \).

Problem Type 10.2b: Set-up, but do not evaluate, an integral that represents the length of the curve

\[ x = f(t) \quad , \quad y = g(t) \quad ; \quad a \leq t \leq b \quad , \]
where \( f(t), \ g(t) \) are expressions in \( t \) and \( a \) and \( b \) are some numbers \((a < b)\).

**Example Problem 10.2b**: Set-up, but do not evaluate, an integral that represents the length of the curve

\[
x = 1 + e^t, \quad y = t^2 \quad ; \quad -3 \leq t \leq 3.
\]

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<th>Steps</th>
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<tr>
<td>1. Compute ( \frac{dy}{dt} ) and ( \frac{dx}{dt} ), getting expressions in ( t ).</td>
<td>1. ( \frac{dx}{dt} = e^t, \ \frac{dy}{dt} = 2t. )</td>
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| 2. Use the formula for the arclength \[
\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \ dt
\] | 2. The arclength is \[
\int_{-3}^{3} \sqrt{(e^t)^2 + (2t)^2} \ dt = \int_{-3}^{3} \sqrt{e^{2t} + 4t^2} \ dt.
\] This is the **answer**! You were not asked to evaluate the integral. |