Solutions to "QUIZ" for Sept. 4, 2008

1. What are: (a) $\sin 0$ (b) $\cos 0$ (c) $\ln 1$.

Sol. to 1.

(a) $\sin 0 = 0$

(b) $\cos 0 = 1$

(c) $\ln 1 = 0$.

Common Mistakes: Everyone got (a) and (b) right, but quite a few people messed up (c). Common wrong answers were 1 and e. It is true that $\ln e = 1$ and $e^1 = e$, but it is a **very basic and important** fact that $\ln 1 = 0$. Those who got it wrong, please write ten times

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2. What is the equation of the line joining $(1, 1)$ and $(2, 3)$?

Sol. to 2. The slope is $m = (3 - 1)/(2 - 1) = 2/1$, and the equation is $(y - 1) = 2(x - 1)$ (also OK is $(y - 3) = 2(x - 2)$). Using algebra we get $y - 1 = 2x - 2$, and finally $y = 2x - 1$.

Ans. to 2.: $y = 2x - 1$.

Comments: Most, but **not** all people know how to do it, but about %10 messed up the algebra somewhere. One student did $m = (2 - 1)/(3 - 1)$. Be very careful to check every step, and try not to make careless algebra mistakes. If you are shaky, you should try to do such problems on your own (and come to free tutoring!).

3. If $f(x) = \frac{1}{x+1}$, what is the inverse function $f^{-1}(x)$?.

Sol. to 3.

Step 1: Replace $f(x)$ by y

$$y = \frac{1}{x+1} \quad .$$

Step 2: Use algebra to solve for x in terms of y.

2a. Multiplying both sides by $x + 1$ we get

$$y(x + 1) = 1 \quad .$$

2b. Opening-up parentheses,

$$yx + y = 1$$

2c. Moving all terms involving x (if applicable) to the left side, and all terms **not** involving x to the right side

$$yx = 1 - y$$

2d. Isolate x :

$$x = \frac{1-y}{y}$$

What is on the right side is $f^{-1}(y)$, so a **correct**, but not quite final answer is

$$f^{-1}(y) = \frac{1-y}{y} \quad .$$

Finally, to conform to the convention that the argument of functions in calculus is traditionally x, change the letter y to the letter x.

$$f^{-1}(x) = \frac{1-x}{x} \quad .$$

Ans. to 3:

$$f^{-1}(x) = \frac{1-x}{x} \quad .$$

Note: Equally correct is

$$f^{-1}(x) = \frac{1}{x} - 1$$

Some common errors

1. Do a "shortcut" and just replace x by y right away getting $f^{-1}(y) = 1/(y+1)$. Of course this is **wrong!** . You need to do the algebra. There are no shortcuts.

2. People knew how to do it, in principle, but made careless algebra mistakes. If you are weak or rusty in algebra, you should practice more! In the free tutoring session LSH A-143 MTh 7:45-8:35am we will review all the needed algebra.

3. **Bad Conceptual Error:** Some people did the algebra correctly, but they messed up at the **bottom line**, and wrote

$$f^{-1}(y) = \frac{1}{x} - 1$$

Or

$$f^{-1}(x) = \frac{1}{y} - 1$$

This is **gibberish**, and I will take off most of the points, since this is a **conceptual error**. You **either** *only* use x (preferably) **or** *only* y, but you can't mix them up!

4. Some people did it in a different way then the way I did it in class. That's OK, as long as you **consistently** do it with that other method. In that method you **first** interchange x and y, and then solve for y (rather than x). Here is how to do it that way:

Step 1: Interchange x and y:

$$x = \frac{1}{y+1} \quad .$$

Step 2: Solve for y, in terms of x.

$\frac{1}{x} = y + 1$ (taking reciprocals), giving $y = \frac{1}{x} - 1$

Ans.: $f^{-1}(x) = \frac{1}{x} - 1$.

That's **perfectly OK** (and, in this problem, even a bit faster). But some people started with that second method, and at the middle switched to my method, solving for x rather than for y, getting the wrong answer.