

Solutions to the ‘QUIZ’ for Sept. 15, 2008

1. Evaluate the limit if it exists:

$$\lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - 3}{x - 2}$$

Solution: First plug-in $x = 2$ and see what happens. You get $0/0$. So this is **indeterminate**, and we must **simplify**. The **conjugate** of $\sqrt{2x+5} - 3$ is $\sqrt{2x+5} + 3$, so we multiply **both** top and bottom by it, and **simplify as much as we can** (correctly!)

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - 3}{x - 2} &= \lim_{x \rightarrow 2} \frac{(\sqrt{2x+5} - 3)(\sqrt{2x+5} + 3)}{(x - 2)(\sqrt{2x+5} + 3)} \\ &= \lim_{x \rightarrow 2} \frac{(\sqrt{2x+5})^2 - 3^2}{(x - 2)(\sqrt{2x+5} + 3)} = \lim_{x \rightarrow 2} \frac{(2x + 5) - 9}{(x - 2)(\sqrt{2x+5} + 3)} \\ &= \lim_{x \rightarrow 2} \frac{2x - 4}{(x - 2)(\sqrt{2x+5} + 3)} \\ &= \lim_{x \rightarrow 2} \frac{2(x - 2)}{(x - 2)(\sqrt{2x+5} + 3)} = \lim_{x \rightarrow 2} \frac{2}{\sqrt{2x+5} + 3} \end{aligned}$$

Now you try your luck again, and plug-in $x = 2$, getting

$$\frac{2}{\sqrt{2 \cdot 2 + 5} + 3} = \frac{2}{\sqrt{9} + 3} = \frac{2}{3 + 3} = \frac{2}{6} = \frac{1}{3} \quad .$$

Ans. to 1.: $\frac{1}{3}$.

Comments: Only about %25 got it perfectly. Another %25 started correctly but then messed up the rather complicated algebra (by making it more complicated than it is, for example, by trying to “simplify” the bottom. Leave the bottom alone! (until the very end)). **PLEASE** come to the free tutoring, this coming Thurs. 7:45AM, LSH A-143, where I will go over it in great detail. This kind of problems is very important.

2. Evaluate

$$\lim_{x \rightarrow 0} \frac{\sin 10x}{5x} \quad .$$

Solutions.

1.: Official Way: “wishful thinking” and then correcting (in order to make $1/5$ to “something” times $1/10$ that “something” must be 2.

$$\lim_{x \rightarrow 0} \frac{\sin 10x}{5x} = 2 \lim_{x \rightarrow 0} \frac{\sin 10x}{10x} = 2 \lim_{10x \rightarrow 0} \frac{\sin 10x}{10x} = 2 \cdot 1 = 2 \quad .$$

2. Shortcut way: By “Dr. Z.’s Get-Rid-Of-Sin Rule” (since what’s inside the sine function ($10x$) is zero at $x = 0$, it is applicable)

$$\lim_{x \rightarrow 0} \frac{\sin 10x}{5x} = \lim_{x \rightarrow 0} \frac{10x}{5x} = \lim_{x \rightarrow 0} 2 = 2 \quad .$$

Ans. to 2: 2.

Comments: Everyone got it right **except** that a few people left the answer as $10/5$. That’s very impolite! You should simplify the answer in such simple division.