Solutions to the "QUIZ" for Nov. 25, 2008

1. Compute

$$\int e^{\cos 2x} \sin 2x \, dx \quad .$$

Sol. to 1: The natural substitution is $u = \cos 2x$ (note: **not** $u = 2x$). Differentiating, we get (by the chain rule)

$$\frac{du}{dx} = -2 \sin 2x \quad ,$$

So here is the "dictionary"

$$u = \cos 2x \qquad , \qquad dx = \frac{-du}{2 \sin 2x} \quad .$$

Performing the translation, we get:

$$\int e^u \sin 2x \left(\frac{-du}{2 \sin 2x} \right)$$

Lo and behold, the x-stuff disappears, and we get that this equals

$$- \int e^u \left(\frac{du}{2} \right) = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u$$

Going back to the x-language, this equals:

$$-\frac{1}{2} e^{\cos 2x}$$

and finally, add $+C$. **Ans. to 1**:

$$-\frac{1}{2} e^{\cos 2x} + C \quad .$$

Comments: Most people got it right, but quite a few people lost the 2 either when they did du/dx, or later.

2. Compute

$$\int_0^{\pi/4} \sin^4 2x \cos 2x \, dx \quad .$$

Sol. to 2: First, let's rewrite this as:

$$\int_0^{\pi/4} (\sin 2x)^4 \cos 2x \, dx$$

The natural candidate for u is what's inside the power, namely

$$u = \sin 2x \quad .$$

Now differentiate:

$$\frac{du}{dx} = 2\cos 2x \quad .$$

Cross multipling, we get

$$dx = \frac{du}{2\cos 2x} \quad .$$

Our "dictionary" is:

$$u = \sin 2x \qquad , \qquad dx = \frac{du}{2\cos 2x} \quad .$$

Since this is a **definite integral**, it is a good idea to also find the limit-of-integration in the u-language. When $x = 0$, $u = \sin(2 \cdot 0) = \sin 0 = 0$. When $x = \pi/4$, $u = \sin(2 \cdot \pi/4) = \sin \pi/2 = 1$. Doing the complete translation, we get

$$\int_0^{\pi/4} (\sin 2x)^4 \cos 2x \, dx = \int_0^1 (u)^4 \cos 2x \, \frac{du}{2\cos 2x} = \frac{1}{2}\int_0^1 u^4 \, du =$$

$$\left.\frac{u^5}{10}\right|_0^1 = \frac{1^{10} - 0^{10}}{10} = \frac{1}{10} \quad .$$

Ans. to 2: $\frac{1}{10}$.

Comment: Another way of doing it is to just do the "anti-derivative", i.e. indefinite integral with respect to x first, and then plug-in the original limits. You would get

$$\int_0^{\pi/4} (\sin 2x)^4 \cos 2x \, dx = \left.\frac{(\sin 2x)^5}{10}\right|_0^{\pi/4} = \frac{1^{10} - 0^{10}}{10} = \frac{1}{10} \quad .$$

The same thing, of course.

Further Comments: Only about a half of the people got it completely right. Some people picked the wrong u (for example $u = 2x$ or $u = \sin^4 2x$, neither of them succeed). Some people forgot to translate the limit-of-integration to the u-langauge and plugged in $u = \pi/4$ and $u = 0$ instead of $u = 1$ and $u = 0$. So watch out!