Solutions to the “QUIZ” for Nov. 25, 2008

1. Compute
\[ \int e^{\cos 2x} \sin 2x \, dx . \]

Solu. to 1: The natural substitution is \( u = \cos 2x \) (note: not \( u = 2x \)). Differentiating, we get (by the chain rule)
\[ \frac{du}{dx} = -2 \sin 2x , \]
So here is the “dictionary”
\[ u = \cos 2x , \quad dx = \frac{-du}{2 \sin 2x} . \]
Performing the translation, we get:
\[ \int e^u \sin 2x \left( \frac{-du}{2 \sin 2x} \right) \]
Lo and behold, the \( x \)-stuff disappears, and we get that this equals
\[ - \int e^u \left( \frac{du}{2} \right) = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u \]
Going back to the \( x \)-language, this equals:
\[ -\frac{1}{2} e^{\cos 2x} \]
and finally, add +C. Ans. to 1:
\[ -\frac{1}{2} e^{\cos 2x} + C . \]

Comments: Most people got it right, but quite a few people lost the 2 either when they did \( du/dx \), or later.

2. Compute
\[ \int_0^{\pi/4} \sin^4 2x \cos 2x \, dx . \]

Solu. to 2: First, let’s rewrite this as:
\[ \int_0^{\pi/4} (\sin 2x)^4 \cos 2x \, dx \]
The natural candidate for \( u \) is what’s inside the power, namely
\[ u = \sin 2x . \]
Now differentiate:
\[ \frac{du}{dx} = 2 \cos 2x . \]

Cross multiplying, we get
\[ dx = \frac{du}{2 \cos 2x} . \]

Our “dictionary” is:
\[ u = \sin 2x , \quad dx = \frac{du}{2 \cos 2x} . \]

Since this is a **definite integral**, it is a good idea to also find the limit-of-integration in the \( u \)-language. When \( x = 0 \), \( u = \sin(2 \cdot 0) = \sin 0 = 0 \). When \( x = \pi/4 \), \( u = \sin(2 \cdot \pi/4) = \sin \pi/2 = 1 \).

Doing the complete translation, we get
\[
\int_{0}^{\pi/4} (\sin 2x)^4 \cos 2x \, dx = \int_{0}^{1} (u)^4 \cos 2x \frac{du}{2 \cos 2x} = \frac{1}{2} \int_{0}^{1} u^4 \, du = \frac{u^5}{10} \bigg|_{0}^{1} = \frac{1^{10} - 0^{10}}{10} = \frac{1}{10} .
\]

**Ans. to 2:** \( \frac{1}{10} \).

**Comment:** Another way of doing it is to just do the “anti-derivative”, i.e. indefinite integral with respect to \( x \) first, and then plug-in the original limits. You would get
\[
\int_{0}^{\pi/4} (\sin 2x)^4 \cos 2x \, dx = \frac{10}{10} (\sin 2x)^5 \bigg|_{0}^{\pi/4} = \frac{1^{10} - 0^{10}}{10} = \frac{1}{10} .
\]

The same thing, of course.

**Further Comments:** Only about a half of the people got it completely right. Some people picked the wrong \( u \) (for example \( u = 2x \) or \( u = \sin^4 2x \), neither of them succeed). Some people forgot to translate the limit-of-integration to the \( u \)-language and plugged in \( u = \pi/4 \) and \( u = 0 \) instead of \( u = 1 \) and \( u = 0 \). So watch out!