Solutions to the "QUIZ" for Nov. 24, 2008

1.

$$\int_1^4 \left(3\sqrt{x} + \frac{1}{x^3} + \frac{1}{\sqrt{x}}\right) d\,x$$

Sol. to 1: First, translate it into power notation:

$$\int_1^4 \left(3x^{1/2} + x^{-3} + x^{-1/2}\right)\, dx$$

Now, integrate, piece-by-piece, using

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad .$$

(except, for this kind of problems, you don't bother with the $+C$). So we have

$$\int_1^4 \left(3x^{1/2} + x^{-3} + x^{-1/2}\right)\, dx =$$

$$3\frac{x^{3/2}}{3/2} + \frac{x^{-2}}{-2} + \frac{x^{1/2}}{1/2}\Big|_1^4$$

Simplifying, we have

$$\left[2(\sqrt{x})^3 - \frac{1}{2x^2} + 2\sqrt{x}\right]\Big|_1^4$$

Now, we plug-in $x = 4$ and $x = 1$ and subtract:

$$\left[2(\sqrt{4})^3 - \frac{1}{2\cdot 4^2} + 2\sqrt{4}\right] - \left[2(\sqrt{1})^3 - \frac{1}{2\cdot 1^2} + 2\sqrt{1}\right] =$$

$$[16 - \frac{1}{32} + 4] - [2 - \frac{1}{2} + 2] = 16 - \frac{1}{32} + \frac{1}{2} = 16\frac{15}{32} = \frac{527}{32} \quad .$$

Ans. to 1: $\frac{527}{32}$.

Comments: Most people did it the right way, but very few people figured out all the fractions correctly, so only a few people got the right answer in simplified form. Many people left it unsimplified, which is usually OK (in such complicated problems).

2. If

$$h(x) = \int_0^x (t+2)^5 (t-1)^{20}\, dt \quad ,$$

what are the critical points of $h(x)$?

Sol. of 2: First we find $h'(x)$, using the **Fundamental Theorem of Calculus**.

$$h'(x) = \frac{d}{dx}\int_0^x (t+2)^5(t-1)^{20} = (x+2)^5(x-1)^{20} \quad .$$

Next, we solve $h'(x) = 0$:

$$(x+2)^5(x-1)^{20} = 0 \quad ,$$

getting $x = -2$ and $x = 1$.

Ans. to 2: The critial points of $h(x)$ are $x = -2$ and $x = 1$.