Solutions to the "QUIZ" for Nov. 17, 2008

1. Use the Linear Approximation to approximate $\sqrt{49.1}$.

Solution to 1: We are supposed to use the formula

$$L(x) = f(a) + f'(a)(x - a) \quad ,$$

First, we have to decide on $f(x)$. This is $f(x) = \sqrt{x} = x^{1/2}$. Second, we have to decide on a. Since $\sqrt{49} = 7$ is "nice", it is natural to take $a = 49$.

Next we find $f'(x)$.

$$f'(x) = (1/2)x^{-1/2} = \frac{1}{2\sqrt{x}}$$

Now, we plug-in for a in the above general formula for $L(x)$:

$$L(x) = f(49) + f'(49)(x - 49) \quad ,$$

We compute

$$f(49) = \sqrt{49} = 7 \quad , \quad f'(49) = \frac{1}{2\sqrt{49}} = \frac{1}{14}$$

Going back above, this gives us:

$$L(x) = 7 + \frac{1}{14}(x - 49) \quad .$$

This is the **linearization**. Finally, to get an approximation for $f(49.1)$, we plug-in $x = 49.1$:

$$\sqrt{49.1} = f(49.1) \approx L(49.1) = 7 + \frac{1}{14}(49.1 - 49) =$$

$$7 + \frac{1}{14} \cdot \frac{1}{10} = 7 + \frac{1}{140} = \frac{981}{140} \quad .$$

Ans. to 1: $\frac{981}{140}$

Comments: Only about half of the people got it completely right. Some people messed up at the end and plugged-in $x = 0.1$ rather than $x = 49.1$. They got confused with the other version, $\Delta f = f'(a)\Delta x$. It is safer to do it the way I did it above.

2. Find

$$\int \frac{x^3 + x}{2x}\, dx.$$

Solution to 2. First Use Algebra!!!.

$$\frac{x^3 + x}{2x} = \frac{x^3}{2x} + \frac{x}{2x} = \frac{x^2}{2} + \frac{1}{2} \quad .$$

Only now you do the integration (anti-derivative)

$$\int \frac{x^3+x}{2x}\,dx = \int (\frac{x^2}{2} + \frac{1}{2})\,dx = \frac{x^3}{6} + \frac{x}{2} + C \quad .$$

Ans. to 2: $\frac{1}{6}x^3 + \frac{1}{2}x + C$.

Comments: Quite a few people didn't realize that they had to use algebra first, and made very serious mistakes by making up their own "rules". The most common one being the "division rule", that lead them to do the following **WRONG** step:

$$\int \frac{x^3+x}{2x}\,dx = \frac{\int (x^3+x)\,dx}{\int (2x)\,dx} \quad .$$

This is **WRONG WRONG WRONG**. So please be careful.