Solutions to the "QUIZ" for Nov. 13, 2008

1. Find

$$\int_{-10}^{10} \sqrt{100-x^2}\,dx \quad .$$

Solution to 1: By the area definition of the definite integral, this is the area under the curve $y = \sqrt{100-x^2}$, above the interval $[-10, 10]$. But this is a semi-circle of radius $\sqrt{100} = 10$, and we all know that the area of a circle of radius r is πr^2, so the area of a semi-circle is half that: $\pi r^2/2$. In our case $r = 10$, so the integral equals $\pi(10)^2/2 = 50\pi$.

Ans. to 1: 50π.

Comments: Most people got it right. A few people took the wrong radius (some took it to be 100, other took it to be 20, I am not sure why). Some people left it as $100\pi/2$. Come-on people, you should be able to do $100/2$ without a calculator.

2. Using graphic analysis, find the definite integral

$$\int_{-1}^{1} \frac{x^5}{x^4+3x^2+1}\,dx$$

(You must justify your answer).

Solution to 2. There is no way that you can do such a complicated integral the "usual" way, and in fact you are told to use graphical considerations.

In such cases, the answer is almost surely going to be 0, but you have to prove it.

We have to check two things. (i) The limits of integration are negatives of each other. It is true in this case since $\{-1, 1\}$ are negatives of each other.

(ii) The integrand

$$f(x) = \frac{x^5}{x^4+3x^2+1} \quad ,$$

is an odd function, which means that $f(-x) = -f(x)$.

Plugging-in $x- > -x$, gives

$$f(-x) = \frac{(-x)^5}{(-x)^4+3(-x)^2+1} = \frac{(-1)^5 \cdot x^5}{(-1)^4(x)^4+3(-1)^2(x)^2+1} =$$

$$\frac{-x^5}{x^4+3x^2+1} = -\frac{x^5}{x^4+3x^2+1} = -f(x) \quad .$$

She function is indeed odd, and the second condtion is satistified.

So the “positive” area is equal to the “negative” area, and their differene is 0.

Ans. to 2: 0 (because the integrand is an odd function and the limit of integrations are negatives of each other).