Solutions to the "QUIZ" for Dec. 8, 2008

1. Verify that the function

$$f(x) = x^4 + 1$$

satisfies the hypothesis of the Mean Value Theorem on the closed interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

Sol. to 1.: $f(x)$ is a **polynomial** so it is automatically continuous and differentiatble, so the MVT applies. MVT promises a number c in the interval $[a, b]$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad .$$

Here $a = 0, b = 2$, $f(x) = x^4 + 1$. So $f'(x) = 4x^3$ and we have to solve

$$4c^3 = \frac{f(2) - f(0)}{2 - 0} = \frac{17 - 1)}{2 - 0} = 8 \quad .$$

Dividing both sides by 4, we get

$$c^3 = 2$$

whose only solution is $c = \sqrt[3]{2}$. It is obviously in our interval $[0, 2]$.

Ans. to 1: $c = \sqrt[3]{2}$.

2. Find the absolute maximum and minimum values of $f(x) = x^4 - 4x + 1$ on the interval $[0, 2]$.

Sol. of 2: $f'(x) = 4x^3 - 4 = 4(x^3 - 1)$. Solving $f'(x) = 0$ yields $x^3 - 1$ whose solution is $x = 1$. Since it lies in $[0, 2]$, we keep it. The finalists are the endpoints $x = 0, x = 2$, and $x = 1$. Plugging-in into $f(x)$, we get $f(0) = 1, f(1) = -2, f(2) = 9$. The **abs. max. value** is 9 at $x = 2$, and the **abs. min value** is -2 at $x = 1$.

Ans. to 2: Abs. min value is -2 (at $x = 1$), Abs. max value is 9 at $x = 2$.