Solutions to the “QUIZ” for Dec. 1, 2008

1. Compute:
\[ \int_{0}^{3} \frac{dx}{x^2 + 9} \].

Sol. to 1: We make the substitution \( x = 3u \). Then \( dx = 3du \) and don’t forget to translate the limits! When \( x = 0, u = 0 \), when \( x = 3, u = 1 \). We have:
\[
\int_{0}^{3} \frac{dx}{x^2 + 9} = \int_{0}^{1} \frac{3du}{(3u)^2 + 9} = \\
\int_{0}^{1} \frac{3du}{9u^2 + 9} = \int_{0}^{1} \frac{3du}{9(u^2 + 1)} = \frac{1}{3} \int_{0}^{1} \frac{du}{u^2 + 1} = \frac{1}{3} \left( \tan^{-1} u \right|_{0}^{1} = \\
\frac{1}{3} (\tan^{-1} 1 - \tan^{-1} 0) = \frac{1}{3} \left( \frac{\pi}{4} - 0 \right) = \frac{\pi}{12} \).
\]
Ans. to 1: \( \frac{\pi}{12} \).

Comments: Many people forgot to transform the limits of integration (0 and 3) from the \( x \)-language to the \( u \)-language.

2. Compute
\[ \int_{0}^{2} 5^x \, dx \].

Sol. to 2:

The easiest way is to use the formula
\[ \int a^x \, dx = \frac{a^x}{\ln a} + C \].

We have:
\[
\int_{0}^{2} 5^x \, dx = \frac{5^2}{\ln 5} \bigg|_{0}^{2} = \frac{5^2}{\ln 5} - \frac{5^0}{\ln 5} = \\
\frac{25}{\ln 5} - \frac{1}{\ln 5} = \frac{24}{\ln 5} .
\]
Ans. to 2): \( \frac{24}{\ln 5} \).

Comments: Many people multiplied by \( \ln 5 \) instead of dividing. When you differentiate, you multiply:
\[ (5^x)' = (\ln 5)5^x \].
But when you anti-differentiate you divide:

$$\int 5^x \, dx = \frac{5^x}{\ln 5} + C .$$

A few people did:

$$\int 5^x \, dx = \frac{5^{x+1}}{x+1} + C .$$

This is **wrong wrong wrong**. Don’t confuse

$$\int x^5 \, dx ,$$

where the $x$ is at the base and the power is 5 with

$$\int 5^x \, dx$$

where the base is the number 5 and the power (exponent) is $x$. 