NAME:____________________________________ **Section:**__________

MATH 151(07-09,20-22), Dr. Z., **Third Practice Final**, Due Dec. 15, 2008, 3:50pm.

1. Write the definition of the derivative $f'(x)$ as a limit and use this definition to find the derivative $f'(2)$ of $f(x) = \frac{1}{x}$. Do not use any rule of differentiation. Be sure to show your work.

2. (a) Assume f is continuous on $[5, 7]$, with the property that $\int_2^3 f(2x+1)\,dx = 10$. Find $\int_5^7 f(x)\,dx$.

(b) Find $\int_0^2 f(x)$ when

$$f(x) = \begin{cases} 2x - 1, & \text{if } x \leq 1; \\ x^2, & \text{if } x \geq 1 \ . \end{cases}$$

3. (a) Use trig identities to evaluate

$$\int (\cos^2 x + \sin^2 x)\, dx$$

and

$$\int (\cos^2 x - \sin^2 x)\, dx$$

Use this to evaluate $\int \cos^2 x\, dx$ and $\int \sin^2 x\, dx$.

(b) Find the left endpoint, midpoint, and right endpoint approximations, L_4,M_4,R_4, for $y = \frac{1}{x+1}$ on $[0, 1]$.

4. (a)Evaluate

$$\sum_{k=1}^{n}(3k-1)^2 = 2^2+5^2+8^2+11^2+\ldots+(3n-1)^2 \quad .$$

(Reminders: $\sum_{k=1}^{n} = n(n+1)/2, \quad ,\sum_{k=1}^{n} k^2 = n(n+1)(2n+1)/6)$.

b) Assume f is an even continuous function on $[-2,2]$. Explain why we have the identities:

$$\int_{-2}^{2} f(x)\,dx = 2\int_{0}^{2} f(x)\,dx \quad ,$$

and

$$\int_{-2}^{2} x^5 f(x)\,dx = 0 \quad .$$

5. Evaluate

a)

$$\int \tan^3 x \sec^2 x \, dx \quad .$$

b)

$$\frac{d}{dx} \int_0^x \sqrt{2+t^3}\, dt \quad and \quad \frac{d}{dx} \int_x^0 \sqrt{2+t^3}\, dt \quad .$$

c)

$$\frac{d}{dx} \int_x^{x^2} e^{z^3} \, dz \quad .$$

6. (a) Evaluate

$$\lim_{x\to 0}\left(\frac{cscx}{x}-\frac{\cot x}{x}\right) \quad .$$

b)

$$\lim_{x\to 0}\left(\frac{cscx}{x}-\frac{\cos x\cot x}{x}\right) \quad .$$

7.Find the point on the curve $y = \sqrt{\ln x}$, $x > 1$ which is closest to the point $(2, 0)$.

8. A farmer wants to fence an area of 16000 square feet and then divide it into 4 parts by placing 3 parallel fences parallel to one of the sides of the rectangle. How would he do as to minimize the cost of the fence?

9. Let

$$f(x) = 2x^3 - 3x^2 + 1$$

a) Determine the intervals on which f is increasing and the intervals on which f is decreasing. Also Determine the local extrema.

b) Determine the intervals on which f is concave up, f is concave down, and determine the points of inflection.

c) Sketch the graph.

10. For the function $f(x) = x^3 \ln x$, defined on $0 < x < \infty$, find the local maxima, the local minimia, the intervals where it is increasing, the intervals where it is decreasing, the inflection points, the intervals where it is concave up, the intervals where it is concave down. Only mention the x coordinates.

11. (a)Assume that $f(x)$ is a differentiable function such that $f(2) = 3$ and $f'(x) \leq -4$ for all x. What can we say about $f(5)$?

(b) Find the horizontal asymptotes of

$$f(x) = \frac{\sqrt{1 + 16x^2}}{x}$$

(c) Find the horizontal asymptotes of

$$f(x) = \frac{|3 + 2x|}{x} \quad .$$

12. Differentiate the following functions, do not simplify.

(a)

$$\tan^{-1}(x^4 + 1)$$

(b)

$$f(x) = \log_{10}(x^5 + e^{2x})$$

(c)

$$f(x) = \frac{x}{\sqrt{x^2 + 4}}$$

13. Suppose that $y = f(x)$ is given implicitly by

$$x^2y^3 + x^3y = 12 \quad .$$

Assume $f(2) = 1$. Find $f'(2)$.

(b) Find $\frac{dy}{dx}$ if

$$2xy^3 + x^2y^2 + (2x+1)y + x^3 = 100 \quad .$$

14. A water trough is of length 100 meters and a cross-section has the shape of an isosceles trapezoid that has width 3 meters at the bottom, 8 meters at the top, and height 5 meters. If the trough is being filled with water at a rate of 2 m^3/min, how fast is the water level rising when the water had depth 3 meters?

15. (a) Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 9x^2 + 12x$ on $[0, 3]$

(b) Find the horizontal and vertical asymptotes of

$$f(x) = \frac{5 - 4x^2 - x^4}{(1 - x^2)(4 - x^2)} \quad .$$