NAME:______________________________ **Section:**________

MATH 151(07-09,20-22), Dr. Z., **Second Practice Final**, Due Dec. 15, 2008, 3:50pm.

1. Write the definition of the derivative $f'(x)$ as a limit and use this definition to find the derivative $f'(3)$ of $f(x) = \frac{3}{x-2}$. Do not use any rule of differentiation. Be sure to show your work.

2. (a) Let $f(x) = e^x g(x)$, and $g(\ln 2) = 4$ and $g'(\ln 2) = -3$. Find $f'(\ln 2)$.

(b) Let $f(x) = \ln(g(x))$. If $g(3) = 10$ and $g'(3) = 20$, find $f'(3)$.

3. (a) For what values of the constant c is the function continuous for all x.

$$f(x) = \begin{cases} cx^2 + 1, & \text{if } x < 2; \\ cx^3, & \text{if } x \geq 2 \text{ .} \end{cases}$$

(b) For what values of the constants c and d is the function continuous and differentiable for all x.

$$f(x) = \begin{cases} 2cx + d, & \text{if } x < 1; \\ cx^2 + (2d+1)x, & \text{if } x \geq 1 \text{ .} \end{cases}$$

4. Let $f(x) = \sqrt{x+5}$.

a) Using the linear approximation of $f(x)$ at $a = 20$, compute an approximation for $f(19)$.

b) Use f''(concavity) to determine whether your approximation is larger or smaller than the true value of $f(19)$.

5. Evaluate the given limits:

a)

$$\lim_{x \to +\infty} \frac{2x^3 + 5x^2 + x - 1}{e^{2x} - x}$$

b)

$$\lim_{x \to 0^+} x^3 \ln \sqrt{x}$$

c)

$$\lim_{x \to +\infty} \frac{8e^{2x} + 5e^x + 1}{4e^{2x} + 15e^x + 23}$$

6. a) Find the absolute maximum and the absolute minimum of

$$f(x) = x^{\frac{2}{3}} - 2x^{\frac{1}{3}}$$

for $-8 \leq x \leq 27$.

b) Find the absolute maximum and the absolute minimum of

$$f(x) = \frac{x}{e^x} \quad for \quad 0 \leq x \leq 2 \quad .$$

7. (a) In the right triangle ΔABC, the right angle is at C and he legs are $|AC| = 12$ and $|BC| = 4$. A rectangle is to be placed inside the triangle. with one corner at C and the opposite corner on the hypotenuse. What are the dimensions of such a rectangle with largest are?

(b) What is the largest possible area of a rectangle that has one of its vertices at the origin $(0, 0)$ and its opposite vertex on the curve $y = 3 - x^2$.

8. (a) Find $y = y(x)$ if $\frac{d^2y}{dx^2} = 8x$, $\frac{dy}{dx}(0) = 2$ and $y(0) = 0$.

(b) Find $y = y(x)$ if $\frac{d^2y}{dx^2} = 2\sin x + \cos x$, $\frac{dy}{dx}(\pi/2) = 0$ and $y(\pi/2) = 1$.

9. Let

$$f(x) = \frac{4}{9 - x^2}$$

a) Find the horizontal and vertical asymptotes of the graph of $f(x)$.

b) Determine the intervals on which f is increasing and the intervals on which f is decreasing. Also Determine the local extrema.

c) Determine the intervals on which f is concave up, f is concave down, and determine the points of inflection.

d) Sketch the graph.

10. (a) Find the equation of the tangent line at the point $(2, 1)$ to the curve defined by the equation

$$x^2 - 2xy - y^2 - 3y = -4$$

(b) Find the equation of the tangent line at the point $(2, 3)$ to the curve defined by the equation

$$e^{xy} + xy = e^6 + 6 \quad .$$

11. Suppose that $f(x)$ is differentiable everywhere and we know that $f(-4) = -2$ and $f'(x) \geq 6$ for all x.

(a) What is the least possible value for $f(3)$?

(b) Using a) show that $f(x)$ has a root on $[-4, 3]$.

12. Use rules of differentiation to calculate $f'(x)$ if (Do not simplify your answers).

(a)

$$\frac{e^{3/x}}{\cos(x^2 - x)}$$

(b)

$$f(x) = (\sin^{-1} x)^{\frac{3}{2}} \cdot \ln(\frac{x^2}{x+1}) \quad .$$

(c)

$$f(x) = \sin(e^{x^2}) \quad .$$

13. (a) Evaluate the following limit-Riemamm sums by any method you like

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2i}{n} - \left(\frac{i}{n} \right)^3 \right] \frac{1}{n}$$

(b) Find the area bounded by the graphs of the two functions $y = x^2 - x + 1$ and $y = 2x - 1$

(c) Using graphic analysis, find the definite integral

$$\int_{-3}^{3} \frac{\sin x + x^3}{x^4 + 3} \quad .$$

(You must justify your answer).

14. A spotlight on the ground shines on a wall 200 meters away. A giant 20 meters tall walks from the spotlight to the wall, at a speed of 4 m./sec. ; his path is perpendicular to the wall. Let x be the distance from his feet to the spotlight and let h be the height of the shadow on the wall. Also let θ be the angle of elevation at the spotlight from the horizontal to the top of his head.
(a) Draw a sketch of the problem, and find a formula relating h and x.
(b) When the giant is 40 meters from the wall, find the height of the shadow and the rate of change of the height of the shadow.
(c) What is the rate of change of θ at the time the giant is 40 meters from the wall?

15. a) Find

$$\int \frac{e^{\left(\frac{2}{x^2}\right)}}{x^3}\,dx$$

b) Find

$$\int_0^{\frac{\pi}{4}} \left(x^{7/2} + \cos(2x)\right) dx$$

c) Find

$$\int_{-10}^{10} \sqrt{100 - x^2}\,dx$$

(d) If

$$h(x) = \int_0^x s(s+1)^{40} \quad ,$$

What are the critical points of $h(x)$?