Answers to Dr. Z’s Three Practice Final Exams
Disclaimer: Not responsible for any typos or other errors. Award of $1 for any mistake found.
Note: Sometimes there is more than one “correct” answer, but using algebra, one should be able to go from one correct answer to another.

Answers to First Practice Final
(http://www.math.rutgers.edu/~zeilberg/calc1N/pf1.pdf)

1. \(-\frac{1}{2\sqrt{x+1}}\) (but you had to do it from the definition!)

2. \(-\frac{y^4 - 3x^2y - y + 8x}{x(x^2 + 4y^3 + 1)}\)

3. (a) 9, forward; (b) \(t = -1, t = 1\); (c) 22.

4. (a) \(-\frac{(x + 1)\sin x + \cos x}{(x + 1)^2}\)

   (b) \(\sin x\cos x + x(\cos^2 x - \sin^2 x)\) OR \(\frac{1}{2}\sin 2x + x\cos 2x\)

   (c) \(\frac{3x\cos x - 4 - 4\sin x}{(x - \cos x)^2}\)

   (d) \(\frac{e^{2x}(1 + 2x)}{(1 + x)^2}\)

5. \(y = 2x + 2\)

6. (a) -4 (b) \(\frac{1}{2}\) (c) 1 (d) 3

7. (a) 3, 0, DNE, 9, -3, DNE (b) \(x = 0, x = 3\)

8. Point: (2, 0). Eq. of tangent line: \(y = x - 2\)

9. (a) \(\frac{6(1 - 3x^4)}{(x^4 + 1)^2}\)

   (b) \(5(5t - 2)e^{-5t}\)

   (c) \((2 - x^2)\sin x + 4x\cos x\).

10. H.A.: \(y = 0\); V.A: \(x = -1, x = 1\); Local max: none; Local Min: none; Point of Inflection: (0, 0); increasing: none; decreasing: \((-\infty, -1), (-1, 1), (1, \infty)\). Verbal description: at the extreme left barely touches the x-axis, just a tiny bit below, eventually, as it reaches \(x = -1\) shoots down to \(-\infty\), only to emerge, right after \(x = -1\) from \(\infty\), going down, via the origin, that it the only point of inflection, down down to \(-\infty\) at \(x = 1\), at \(x = 1\)
it suddenly jumps to $\infty$ and starts its descent down, eventually getting very close, and almost parallel to the $x$-axis, but never crossing it.

11. $\frac{41}{49} \, \Omega/s$.

12. (a) $1 - \frac{1}{100e}$ (b) $\frac{5}{3}$

13. (a) $\sqrt{2}$ (b) abs. max: 9 at $x = 2$; abs. min: $-2$ at $x = 1$.

14. inc.: $(-\infty, 1)$, dec.: $(1, \infty)$, (b) local max: $(1, \frac{1}{e})$, local min: none, (c) concave down: $(-\infty, 2)$, concave up: $(2, \infty)$, point of inflection: $(2, \frac{2}{e^2})$. (d) Verbal description: comes from the left from $-\infty$, climbing up, passing through the origin, peaking at the max. $(1, \frac{1}{e})$, where it goes down, passing through the inflection point $(2, \frac{2}{e^2})$, and keep going down, but slower and slower, eventually barely touching the positive $t$-axis.

15. $10 \times 10 \times 10$.

16. a) $\frac{3}{2} x^2 + 5 \sin^{-1} x + C$ (b) $\frac{x^2}{2} + x + \ln |x| + C$ (c) $x^2 + \frac{1}{x} + 1$.

**Answers to Second Practice Final**

(http://www.math.rutgers.edu/~zeilberg/calc1N/pf2.pdf)

1. $-3$ (but you had to do it from the definition!)

2. (a) 2 (b) 2

3. (a) $c = \frac{1}{4}$ (b) $c = \frac{1}{2}$, $d = -\frac{1}{2}$.

4. (a) $\frac{41}{10}$; (b) larger (because curve is concave down at $x = 20$).

5. (a) 0 (b) 0 (c) 2

6. (a) abs. max.: 8 at $x = -8$; abs. min: $-1$ at $x = 1$. (b) abs. min.: 0 at $x = 0$; abs. max: $\frac{1}{e}$ at $x = 1$.

7. (a) 2 by 6 (b) 2

8. (a) $y = \frac{3}{4} x^3 + 2x$ (b) $y = -2 \sin x - \cos x - x + 3 + \frac{\pi}{2}$

9. a) H.A.: $y = 0$, V.A.: $x = -3, x = 3$. b) dec.: $(-\infty, -2), (-2, 0)$ ; inc.: $(0, 2), (2, \infty)$.

local min: $(0, \frac{1}{6})$, local max: none. c): concave up: $(-3, 3)$, concave down: $(-\infty, -3),(3, \infty)$, points of inflection: none. d): you do it!

10. (a) $y = \frac{2}{5} x + \frac{5}{9}$; (b) $y = -\frac{3}{2} x + 6$.

11. (a) 40 (b) Use IVT, $f(-4)$ is neg. while $f(3)$ is pos.

12. (a)

$$\frac{-3e^{\frac{x}{2}} \cos(x^2 - x) + e^{\frac{x}{2}} (2x - 1) \sin(x^2 - x)}{\cos^2(x^2 - x)}$$

(b)

$$\frac{3 \sqrt{\sin^{-1} x (2 \ln x - \ln(x + 1))}}{\sqrt{1 - x^2}} + (\sin^{-1} x)^{3/2} \left( \frac{2}{x} - \frac{1}{x + 1} \right)$$

(c) $2xe^{x^2} \cos(e^{x^2})$

13. (a) $\frac{3}{4}$ (b) $\frac{1}{6}$ (c) 0 (the integrand is an odd function [why?]).
14. (a)\( h = \frac{4000}{\pi} \) (b) height of shadow: 25 meters; rate of change of height of shadow: \(-\frac{5}{8}\) m/sec. (c) \(-\frac{1}{325}\) radians per sec.

15. (a) \(-\frac{1}{4}e^{\frac{x}{3}} + C\) (b) \(\pi^4\sqrt{\pi}/2304 + \frac{1}{2}\) (c) \(50\pi\) (d) \(x = 0\) and \(x = -1\).

Answers to Third Practice Final

(http://www.math.rutgers.edu/~zeilberg/calc1N/pf3.pdf)

1. \(-\frac{1}{4}\) (but you had to do it from the definition).
2. (a) 20 (b) \(\frac{7}{3}\)
3. (a) \(\frac{x}{2} + \frac{\sin 2x}{4} + C\), \(\frac{x}{2} - \frac{\sin 2x}{4} + C\) (b) \(\frac{319}{4320}, \frac{4448}{840}\)
4. (a) \(\frac{1}{3}n(6n^2 + 3n - 1)\) (b) use symmetry.
5. (a) \(\frac{1}{4}\tan^4 x + C\) (b) \(\sqrt{2 + x^3}, -\sqrt{2 + x^3}\) (c) \(2xe^{-x^2} - e^x\)
6. (a) \(\frac{1}{2}\) (b) 1.
7. \((2 + \sqrt{2}, \sqrt{\ln(2 + \sqrt{2})})\)
8. 200 by 80.
9. (a) inc.: \((\infty, 0), (1, \infty)\); dec.: \((0, 1)\). Local max: \((0, 1), local min: (1, 0)\). (b) concave up: \((\frac{1}{2}, \infty)\); concave down: \((\infty, \frac{1}{2})\). Point of inflection: \((\frac{1}{2}, \frac{1}{2})\) (c) You do it!
10. inc.: \((e^{-1/3}, \infty)\); dec.: \((\infty, e^{-1/3})\). Local min: \((e^{-1/3}, -\frac{1}{3e})\), concave down: \((\infty, e^{-5/6})\), concave up: \((e^{-5/6}, \infty)\); p.o.i: \((e^{-5/6}, -\frac{1}{6}e^{-5/2})\).
11. (a) \(f(5) \leq -9\) (b) \(y = 4, y = -4\) (c) \(y = 2, y = -2\).
12. (a) \[
\frac{4x^3}{1 + (x^4 + 1)^2}
\]
(b) \[
\frac{5x^4 + 2e^{2x}}{(\ln 10)(x^5 + e^{2x})}
\]
(c) \[
(x^2 + 4)^{-1/2} - x^2(x^2 + 4)^{-3/2}
\]
13. (a) \(-\frac{4}{3}\) (b) \[
\frac{-2y^3 + 2xy^2 + 2y + 3x^2}{6xy^2 + 2x^2y + 2x + 1}
\]
14. \(\frac{1}{300}\) m/s.
15. (a) abs. max: 9 and \(x = 3\); abs. min: 4 at \(x = 2\). (b) H.A.: \(y = -1\), V.A.: \(x = -1, x = 1, x = -2, x = 2\).