NAME:______________________________________ **Section:**___________

MATH 151(07-09,20-22), Dr. Z., **First Practice Final**, Due Dec. 15, 2008, 3:50pm.

1. Find the derivatives of

$$f(x) = \frac{1}{\sqrt{x+1}}$$

from the definition. [No Credit for other methods].

2. Find dy/dx by implicit differentiation if

$$x^3y + xy + xy^4 = 4x^2 - 4 \quad .$$

3. If the Law of Motion of a particle is $s(t) = t^3 - 3t + 1$, find
(a) The speed and direction (forward or backwards) at $t = 2$.
(b) The time(s) when it is at rest.
(c) The total distance traveled between $t = 0$ and $t = 3$.

4. Differentiate

$$(a) \quad \frac{\cos x}{x+1} \quad ; \quad (b) \quad x \sin x \cos x \quad ;$$

$$(c) \quad \frac{1+3\sin x}{x-\cos x} \quad ; \quad (d) \quad \frac{e^{2x}}{1+x} \quad .$$

5. Find an equation of the tangent line to the curve

$$y = \frac{4}{1 + e^{-2x}} \quad ,$$

at the point $(0, 2)$.

6. Find the limits

$$(a) \quad \lim_{x\to -2} \frac{x^2-4}{x^2+5x+6} \quad ; \quad (b) \lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \quad ;$$

$$(c) \quad \lim_{x\to\infty} \frac{2x^3+5}{(2x+1)(x+3)(x-1)} \quad ; \quad (d) \lim_{x\to 0} \frac{\sin 6x}{\sin 2x} \quad .$$

7. Let

$$f(x) = \begin{cases} \sqrt{-2x}, & \text{if } x < 0; \\ 3 - 2x, & \text{if } 0 \le x < 3; \\ (2x-3)^2, & \text{if } x \ge 3\ . \end{cases}$$

(a) For each of the following limits, evaluate it, if it exists.

(i) $\lim_{x\to 0^+} f(x)$

(ii) $\lim_{x\to 0^-} f(x)$

(iii) $\lim_{x\to 0} f(x)$

(iv) $\lim_{x\to 3^+} f(x)$

(v) $\lim_{x\to 3^-} f(x)$

(vi) $\lim_{x\to 3} f(x)$.

(b) Where is f discontinuous?

8. Find the point on the curve

$$y = x^2 - 3x + 2$$

where the tangent line is parallel to the line $y = x + 4$. Then find the equation of that tangent line.

9. Find the second derivative of

$$(a)\ f(x) = 3\tan^{-1}(x^2) \quad ; \quad (b)\ g(t) = te^{-5t} \quad ; \quad (c)\ h(x) = x^2 \sin x \quad .$$

10. Sketch the function

$$f(x) = \frac{x}{x^2 - 1} \quad ,$$

indicating all asymptotes, local extrema, inflection points, intervals of increase and decrease, and concave up/down.

11. If two resistors with resistances R_1, R_2 are connected in parallel then the total resistance R, measured in ohms (Ω), is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1, R_2 are increasing at rates of $1\ \Omega/s$, $2\ \Omega/s$ respectively, how fast is R changing when $R_1 = 1/3\ \Omega$ and $R_2 = 1/4\ \Omega$.

12.

(a) Use differentials to estimate $\ln(e - .01)$.

(b) Use one step of Newton's method to find an approximation for the root of the equation $x^5 = 2x + 1$, taking $x_1 = 1$.

13. (a) Verify that the function

$$f(x) = x^4 + 1$$

satisfies the hypothesis of the Mean Value Theorem on the closed interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

(b) FInd the absolute maximum and minimum values of $f(x) = x^4 - 4x + 1$ on the interval $[0, 2]$.

14. Consider the function $f(t) = te^{-t}$.
(a) Find the intervals of increase and decrease.

(b) Find the local maximum and local minimum values.

(c) Find the intervals of concavity and inflection points.

(d) Use the above information to sketch the graph.

15. An open box, with square base, is made to have a volume of $1000\,cm^3$. The material for the bottom is twice as expensive as the material for the sides. What are the dimensions that will minimize the total cost of the material?

16. (a) Find the most general antiderivative

$$f(x) = 3x + 5(1 - x^2)^{-1/2}$$

(b) Find the most general antiderivative

$$\frac{x^2 + x + 1}{x}$$

(c) Find f if $f'(x) = 2x - 3/x^4, x > 0, f(1) = 3$.