NAME:____________________________________ **Section:**________

MATH 151(07-09,20-22), Dr. Z. , **Third Practice Exam for the Second Midterm (Nov. 20, 2008)**

Remember: 1. Show all your work. 2. Make sure that the answer(s) is (are) of the right type. If your answer will be of the wrong type (for example, if the answer is supposed to be an equation of a straight line, and your final answer is $y = x^2(x-2)+3$ (which is **not** an equation of straight line) you would get no points at all, even if everything is correct except for one step.

——Do not write below this line—————

1. (out of 16)

2. (out of 12)

3. (out of 12)

4. (out of 12)

5. (out of 12)

6. (out of 12)

7. (out of 12)

8. (out of 12)

TOTAL: (out of 100)

1. (16 points altogether) Consider the function

$$f(x) = \frac{x}{2x-1} \quad .$$

(a) (3 points) Find the horizontal and vertical asymptotes.

(b) (3 points) Find all the local maxima and local minima

(c) (2 points) Find all the inflection points (if they exist)

(d) (4 points) In what intervals is the function (i) increasing? (ii) decreasing(?) (iii) concave up? (iv) concave down?

(e) (4 points) Sketch the graph

2. (a) (4 points) Find

$$\lim_{x \to 0} \frac{x^3}{\sin x - x}$$

(b) (4 points)

$$\lim_{x \to \infty} \frac{3x^3 + 4x^2}{4x^3 - 7}$$

(c) (4 points)

$$\lim_{x \to 0^+} \frac{\ln x}{\sqrt{x}}$$

3. (12 points) If two resistors with resistance R_1 and R_2 are connected in parallel, then the total resistance is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad .$$

If R_1 and R_2 are increasing at a rate of 2 and 3 ohms per sec. respectively, how fast is R changing when $R_1 = 1/2$ and $R_2 = 1/3$.

4. (12 points)
There is a spotlight on the top of a wall 20 meters high. A man 2 meters tall walks away from the wall, at a speed of 1 m./sec. ; his path is perpendicular to the wall. Let x be the distance from his feet to the bottom of the wall and let h be the lenght of the shadow on the ground.
(a) Draw a sketch of the problem, and find a formula relating h and x.
(b) When the man is 6 meters from the wall, find the length of the shadow and the rate of change of the length of the shadow.

5. (12 points) A farmer wants to fence an area of 10000 square feet and then divide it into 2 parts by a parallel fence parallel to one of the sides of the rectangle. What is the smallest possible length of fencing he has to buy, in order to achieve his goal?

6. (12 points) Find the minimal area of a triangle formed in the first quadrant by the x-axis, y-axis, and a tangent line to the graph of $y = 1 - x^2$.

7. (a) (6 points) Find the derivative of the function $f(x) = (1 + \ln x)^{100}(\sin + \cos x)^{50}$.

(b) (6 points) Solve the differntial equation

$$y'''(x) = 6 \quad ,$$

with initial condition $y(1) = 1$, $y'(1) = 3$, $y''(1) = 6$

8.
Reminders: For the definite integral

$$\int_a^b f(x) \quad ,$$

then the **Right-Approximation**, **Left-Approximation** and **Mid-Approximation** with N intervals is (let $h = \frac{b-a}{N}$), are:

$$R_N = h\sum_{j=1}^{N} f(a+jh) \quad , \quad L_N = h\sum_{j=0}^{N-1} f(a+jh) \quad , \quad M_N = h\sum_{j=1}^{N} f(a+(j-\frac{1}{2})h)$$

(a) (4 points) Compute R_4 for $f(x) = 2x - 2$ and the interval $[1, 5]$. Draw a picture of the region that R_4 measures.

(b) (4 points) Compute L_4 for $f(x) = 2x - 2$ and the interval $[1, 5]$. Draw a picture of the region that L_4 measures.

(c) (4 points) Who is larger R_4 or L_4? Explain! (using the above pictures).