NAME:______________________________________ **Section:**_________

MATH 151(07-09,20-22), Dr. Z. , **Second Practice Exam for the Second Midterm (on Nov. 20, 2008)**

Remember: 1. Show all your work. 2. Make sure that the answer(s) is (are) of the right type. If your answer will be of the wrong type (for example, if the answer is supposed to be an equation of a straight line, and your final answer is $y = x^2(x-2)+3$ (which is **not** an equation of straight line) you would get no points at all, even if everything is correct except for one step.
——Do not write below this line——————

1. (out of 16)

2. (out of 12)

3. (out of 12)

4. (out of 12)

5. (out of 12)

6. (out of 12)

7. (out of 12)

8. (out of 12)

TOTAL: (out of 100)

1. (16 points altogether) Consider the function

$$f(x) = \frac{x}{x^2+1}$$

(a) (2 points) Find the horizontal asymptotes

(b) (2 points) Find the vertical asymptotes

(c) (2 points) Find all the local maxima and local minima

(d) (2 points) Find all the inflection points (if they exist)

(e) (4 points) In what intervals is the function (i) increasing? (ii) decreasing(?) (iii) concave up? (iv) concave down?

(f) (4 points) Sketch the graph

2. (a) (6 points) Find

$$\lim_{x \to 0} \frac{x \sin 4x}{(e^x - 1)\sin 11x}$$

(b) (6 points)

$$\lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4}$$

3. (a) (6 points) Use two steps in Newton's method to approximate the root of $3x^2-3x+1 = 0$, starting with $x_0 = 1/3$.

(b) (6 points) Use the linearization of $f(x) = \ln x$ at $x = e^5$ to find an approximation to $\ln(e^5 + 0.1)$.

4. (a) (6 points) The area of a circle is expanding at a rate of 1 square centimeter per second. How fast is its perimeter expanding when the area of the circle happens to be 100π?

(b) (6 points) Find the linearization $L(x)$ of $f(x) = \frac{1}{x^3}$ at $x = 2$.

5. Differentiate the following functions

(a) (4 points)

$$f(x) = x^3(\ln(1+x))^2$$

(b) (4 points)

$$f(x) = 10^{\tan^{-1} x}$$

(c) (4 points)

$$f(x) = e^{\ln((\sqrt{x^3+1})^2)} \quad .$$

6. (12 points) A jeweler has to design a closed box, with a square base, with a budget of 1000 dollars for materials. The cost of the material for the bottom and top is $\frac{5}{3}$ dollars per square inch, while the cost of the material for the four sides is 2 dollars per square inch. What are the dimensions of the box that would **maximize** the volume?

7. (a) (6 points) Find the absolute maximum and the absolute minimum of the function

$$f(x) = x^{4/3} - 2x^{2/3}$$

on the interval $[0, 8]$.

(b) (6 points) Solve the differential equation

$$y'(x) = \sec^2 x + x \quad ,$$

with initial condition $y(\pi/4) = \frac{\pi^2}{32} + 3$,

8. (12 points) Use the definition of the definite integral

$$\int_a^b f(x)dx \;=\; \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\Delta x$$

(where $\Delta x = (b - a)/n$). to evaluate the definite integral

$$\int_0^2 x^3\, dx$$

Hint: You may use one or more of the formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad , \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad , \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2 \quad .$$