NAME:_____ **Section:**_____

MATH 151(07-09, 20-22), Dr. Z. , **First Practice Exam for the Second Midterm (on Nov. 20, 2008)**

Remember: 1. Show all your work. 2. Make sure that the answer(s) is (are) of the right type. If your answer will be of the wrong type (for example, if the answer is supposed to be an equation of a straight line, and your final answer is $y = x^2(x-2)+3$ (which is **not** an equation of straight line) you would get no points at all, even if everything is correct except for one step.
——Do not write below this line————

1. (out of 16)

2. (out of 12)

3. (out of 12)

4. (out of 12)

5. (out of 12)

6. (out of 12)

7. (out of 12)

8. (out of 12)

TOTAL: (out of 100)

1. (16 points altogether) Consider the function

$$f(x) = x^2(2x - 9)$$

(a) (6 points) Find all the local maxima and local minima

(b) (2 points) Find all the inflection points (if they exist)

(c) (4 points) In what intervals is the function (i) increasing? (ii) decreasing(?) (iii) concave up? (iv) concave down?

(d) (4 points) Sketch the graph

2. At a given moment, a plane passes directly above a radar station at an altitude of 6 miles.
(a) (6 points) If the plane's speed is 500 mph, how fast is the distance between the plane and the station changing half an hour later?
(b) (6 points) How fast is the distance between the plane and the station changing when the plane passes directly above the station.

3. (12 points) Find the point on the curve

$$y = x^2 \quad ,$$

closest to the point $(3, 0)$.

4. (12 points) Let $f(x) = \sqrt{1-x}$
(a) (6 points) Using the linear approximation of $f(x)$ at $a = -3$ compute an approximation to $f(-4)$.
(b) (6 points) Use f'' (concavity) to determine whether your approximation is larger or smaller than the true value of $f(-4)$.

5. Differentiate the following functions (**Do not simplify**).
(a) (4 points)

$$f(x) = (\cos x + \sin x) \cdot \ln(\tan^{-1} x)$$

(b) (4 points)

$$f(x) = \log_2(\tan^{-1} x)$$

(c) (4 points)

$$f(x) = e^{\ln \cos x}$$

6. Determine the intervals on which the function is concave up or concave down and find the points of inflection

(a) (6 points) $f(x) = (x^2 - 3)e^x$

(b) (6 points) $f(x) = 4x^5 - 4x^4$

7. (a) Find the derivative of the function $f(x) = (1 + \ln x)^{10}(3 + e^x)^{20}(1 - \sqrt{x})^{100}$.

(b) (6 points) Solve the differntial equation

$$y''(x) = -\cos x + 2 \quad ,$$

with initial condition $y(0) = 2$, $y'(0) = 0$.

8. Find (a) (4 points)

$$\int \frac{x^5 + 2x^3}{x^2}\, dx$$

(b) (4 points)

$$\int x \sin^2 x\, dx + \int x \cos^2 x\, dx$$

(c) (4 points)

$$\int x^2(x-1)\, dx$$