NAME:______________________________________ **Section:**_________

MATH 151(20-22), Dr. Z. , **Second Midterm**, Thursday, Nov. 20, 2008.

Remember: 1. Show all your work. 2. Make sure that the answer(s) is (are) of the right type. If your answer will be of the wrong type (for example, if the answer is supposed to be an equation of a straight line, and your final answer is $y = x^2(x-2)+3$ (which is **not** an equation of straight line) you would get no points at all, even if everything is correct except for one step.
——Do not write below this line——————

1. (out of 16)

2. (out of 12)

3. (out of 12)

4. (out of 12)

5. (out of 12)

6. (out of 12)

7. (out of 12)

8. (out of 12)

TOTAL: (out of 100)

1. (16 points altogether) Consider the function

$$f(x) = \frac{x^2 + 1}{x^2 - 4}$$

(a) (2 points) Find the horizontal asymptotes

(b) (2 points) Find the vertical asymptotes

(c) (2 points) Find all the local maxima and local minima

(d) (2 points) Find all the inflection points (if they exist)

(e) (4 points) In what intervals is the function (i) increasing? (ii) decreasing(?) (iii) concave up? (iv) concave down?

(f) (4 points) Sketch the graph

2. (a) (6 points) Find

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$$

(b) (6 points)

$$\lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6} - \frac{x^5}{120}}{x^7}$$

3. (a) (6 points) Use one step in Newton's method to approximate $\sqrt{15}$, starting with $x_0 = 4$.

(b) (6 points) Use the linearization of $f(x) = \sqrt{x}$ at $x = 16$ to find an approximation to $\sqrt{15}$.

4. (a) (6 points) The volume of a cube is expanding at a rate of 1 cubic centimeter per second. How fast is its surface area expanding when the volume of the cube is 1000 cubic centimeters?

(b) (6 points) Find the linearization $L(x)$ of $f(x) = \ln x$ at $x = e^2$.

5. Differentiate the following functions

(a) (4 points)

$$f(x) = \ln x \cdot \tan^{-1}(x^2 + 1)$$

(b) (4 points)

$$f(x) = 10^{\sin x} 5^{\cos x}$$

(c) (4 points)

$$f(x) = (\sqrt{\ln(e^x) + 1})^2$$

6. (12 points) A jeweler has to design am open box, with a square base, of 2000 cubic centimeters, where the bottom is made of silver, and the four sides are made of gold. The cost of silver is 10 dollars per square centimeter, while the cost of gold is 20 dollars per square centimeter. What are the dimensions of the box that would **minimize** the cost?

7. (a) (6 points) Find the absolute maximum and the absolute minimum of the function

$$f(x) = xe^{-2x}$$

on the interval $[0, 2]$.

(b) (6 points) Solve the differntial equation

$$y''(x) = \sin x \quad ,$$

with initial condition $y(\pi/2) = 1$, $y'(\pi/2) = 2$.

8. (12 points) Use the definition of the definite integral

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

(where $\Delta x = (b-a)/n$ and $x_i = a + i\Delta x$). to evaluate the definite integral

$$\int_0^2 3x dx$$

Hint: You may need the formula:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad .$$