**NAME:**________________________________________ **Section:**_________

MATH 151(07-09), Dr. Z. , **Second Midterm**, Thursday, Nov. 20, 2008.

1. (16 points altogether ) Consider the function

$$f(x) = \frac{x^2+1}{x^2-4}$$

(a) (2 points) Find the horizontal asymptotes

(b) (2 points) Find the vertical asymptotes

(c) (2 points) Find all the local maxima and local minima

(d) (2 points) Find all the inflection points (if they exist)

(e) (4 points) In what intervals is the function (i) increasing? (ii) decreasing(?) (iii) concave up? (iv) concave down?

(f) (4 points) Sketch the graph

2. (a) (6 points) Find

$$\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$$

(b) ( 6 points)

$$\lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6} - \frac{x^5}{120}}{x^7}$$

3. (a) (6 points) Use one step in Newton's method to approximate $\sqrt{15}$, starting with $x_0 = 4$.

(b) ( 6 points) Use the linearization of $f(x) = \sqrt{x}$ at $x = 16$ to find an approximation to $\sqrt{15}$.

4. (a) ( 6 points) A metal cube of volume 1000 cubic centimeters is left on a sunny beach. Heat expands the volume by %1. Approximate the percentage increase in the surface area.

(b) (6 points) Find the linearization $L(x)$ of $f(x) = \ln x$ at $x = e^2$.

5. Differentiate the following functions

(a) (4 points)

$$f(x) = \ln x \tan^{-1}(x^2 + 1)$$

(b) (4 points)

$$f(x) = 10^{\sin x} 5^{\cos x}$$

(c) (4 points)

$$f(x) = (\sqrt{\ln(e^x) + 1})^2$$

6. (12 points) A jeweler has to design am open box, with a square base, of 1000 cubic centimeters, where the bottom is made of silver, and the four sides are made of gold. The cost of silver is 10 dollars per square centimeter, while the cost of gold is 20 dollars per square centimeter. What are the dimensions of the box that would **minimize** the cost?

7. (a) (6 points) Find the absolute maximum and the absolute minimum of the function

$$f(x) = xe^{-2x}$$

on the interval $[0, 2]$.

(b) (6 points) Solve the differntial equation

$$y''(x) = \sin x \quad ,$$

with initial condition $y(\pi/2) = 1$, $y'(\pi/2) = 2$.

8. (12 points) Compute the area under the curve $y = 3x$ over $[0, 3]$ using $\lim_{N\to\infty} R_N$ .