Dr. Z's Math151 Handout #3.7 [The Chain Rule]

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Problem Type 3.7.1 : Write the composite function in the form $f(g(x))$, Identify the inner function $u = g(x)$ and the outer function $y = f(u)$. Then find the derivative dy/dx.

Example Problem 3.7.1: Do the above for $\sin(\tan x)$.

Steps	**Example**
1. Start with the *inner function* (the one that is usually *inside* the parantheses.) This is your $u = g(x)$.	**1.** $u = g(x) = \tan x$.
2. Replace the inner function you found above by u, so $f(g(x))$ becomes $f(u)$.	**2.** Replacing $\tan x$ by u, in $\sin(\tan x)$, we see that $f(u) = \sin(u)$.
3. Use the chain rule $[f(g(x))]' = f'(g(x)) \cdot g'(x) \quad .$	**3.** $f'(u) = \cos(u)$ and $g'(x) = \sec^2 x$, so, by the chain rule $[\sin(\tan x)]' = \cos(\tan x) \sec^2 x$.

Problem Type 3.7.2 :

Find the derivative of the function $(Expression(x))^n$.

Example Problem 3.7.2:

Find the derivative of the function $(1+\cos^3 x)^5$.

Steps

1. The chain rule when the outside function is a power is

$$[Expression(x)^n]' =$$

$$nExpression(x)^{n-1} \cdot Expression'(x) \quad .$$

2. Do the rest of the inside differentiation, using any applicable rules, in particular, you might have to use the chain rule again.

3. Clean up using algebra.

Example

1.

$$[(1+\cos^3 x)^5]' = 5(1+\cos^3 x)^4(1+\cos^3 x)'$$

2.

$$5(1+\cos^3 x)^4(1+\cos^3 x)' = 5(1+\cos^3 x)^4(1'+(\cos^3 x)') =$$

$$5(1+\cos^3 x)^4(0+3(\cos^2 x)(\cos x)') =$$

$$5(1+\cos^3 x)^4(3(\cos^2 x)(-\sin x))$$

3.

$$-15\sin x\cos^2 x(1+\cos^3 x)^4 \quad .$$

Problem from a Previous Final Exam (Spring 2008, #12a (5 points)).

Use the rules of differentiation to calculate $f'(x)$ (Do not simplify your answer).

$$f(x) = \frac{e^{\frac{2}{x}}}{\cos(x^2 - x)}$$

Solution: First we must use the **quotient rule**.

$$f'(x) = \frac{\cos(x^2 - x)' e^{\frac{2}{x}} - \cos(x^2 - x)(e^{\frac{2}{x}})'}{(\cos(x^2 - x))^2}$$

Second, we must use the **chain rule**.

$$= \frac{-\sin(x^2 - x)(2x - 1)e^{\frac{2}{x}} - \cos(x^2 - x)e^{\frac{2}{x}}(-2/x^2)}{(\cos(x^2 - x))^2} \quad .$$

This is the answer!. No one asked you to simplify it.