Dr. Z's Math151 Handout #3.4 [Rates of Change]

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Problem Type 3.4.1 : A particle moves according to a law of motion $s = f(t)$, $t > 0$, where t is measured in some unit of time and s in some unit of distance.

(a) Find the velocity at any given time t.

(b) What is the velocity after t_1 seconds, for some specific number t_1.

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction?

(e) Find the total distance travelled during the first t_2 seconds, for some specific t_2.

Example Problem 3.4.1: A particle moves according to a law of motion $s = t^4 - 2t^2 + 1$, $t > 0$, where t is measured in seconds and s is measured in feet.

(a) Find the velocity at any given time t.

(b) What is the velocity after 3 seconds?

(c) When is the particle at rest?

(d) When is the particle moving in the positive direction?

(e) Find the total distance travelled during the first 8 seconds.

Steps	Example
a. The velocity $v(t)$, is $\frac{ds}{dt}$, in other words the derivative of $s(t)$ with respect to t.	**a.** v(t)=$\frac{ds}{dt} = (t^4 - 2t^2 + 1)' = 4t^3 - 4t$.
b. Plug in $t = t_1$ in $v(t)$.	**b.** $v(3) = 4 \cdot 3^3 - 4 \cdot 3 = 96$.

c. You solve the equation $v(t) = 0$.

c. $v(t) = 4t^3 - 4t = 4t(t-1)(t+1)$. So $v(t) = 0$ when $t = -1, 0, 1$. But we are only talking about $t > 0$, so the particle is at rest at the start ($t = 0$), and after one second ($t = 1$).

d. It is moving in the positive direction when $v(t) > 0$.

d. $v(t) = 4t(t-1)(t+1) > 0$ when $t-1 > 0$, since always $t > 0$ and $t+1 > 0$. That is when $t > 1$. For $0 < t < 1$ $v(t)$ is negative, so between $t = 0$ and $t = 1$ the particle is moving in the *negative direction*.

e. Warning: The total distance is (usually) not $s(t_2)$!, one has to treat separately the periods when the particle was moving backwards and forwards.

e. The distance travelled from $t = 0$ to $t = 1$ is $-(s(1) - s(0)) = -(0 - 1) = 1$. The distance travelled between $t = 1$ and $t = 8$ is $s(8) - s(1) = (8^4 - 2 \cdot 8^2 + 1) - 0$ which equals 3969. So the *total* distance is $1 + 3969 = 3970$ feet. **Ans.**: The total distance travelled during the first 8 seconds is 3970 feet.