Problem Type 3.3.1: Differentiate $f(x) = \text{Expression}_1(x) \times \text{Expression}_2(x)$, where both expressions are ‘easy’ to differentiate from known rules.

Example Problem 3.3.1: Differentiate $f(x) = x^4e^x$.

<table>
<thead>
<tr>
<th>Steps</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Use the product rule $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$</td>
<td>1. $(x^4e^x)' = (x^4)'e^x + (x^4)(e^x)' = 4x^3e^x + x^4e^x$</td>
</tr>
<tr>
<td>2. Use algebra to simplify.</td>
<td>2. Ans.: $(4x^3 + x^4)e^x$.</td>
</tr>
</tbody>
</table>
Problem Type 3.3.2:

Differentiate

\[ f(x) = \frac{\text{Expression}_1(x)}{\text{Expression}_2(x)} . \]

where both expressions are ‘easy’ to differentiate from known rules.

Example Problem 3.3.2: Differentiate

\[ y = \frac{t^3 + t}{t^4 - 2} . \]

Steps

1. Use the quotient rule

\[
\left( \frac{f(x)}{g(x)} \right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} .
\]

Example

1.

\[
\left( \frac{t^3 + t}{t^4 - 2} \right)' = \frac{(t^3 + t)'(t^4 - 2) - (t^3 + t)(t^4 - 2)'}{(t^4 - 2)^2} .
\]

\[
= \frac{(3t^2 + 1)(t^4 - 2) - (t^3 + t)(4t^3)}{(t^4 - 2)^2} .
\]

2. Use algebra to simplify.

Important comment: Sometimes you are told to differentiate a function, and it says “do not simplify”. In that case, you don’t need the present step.

2.

\[
= \frac{(3t^6 - 6t^2 + t^4 - 2) - (4t^6 + 4t^4)}{(t^4 - 2)^2} .
\]

\[
= \frac{3t^6 - 6t^2 + t^4 - 2 - 4t^6 - 4t^4}{(t^4 - 2)^2} .
\]

\[
= \frac{-t^6 - 3t^4 - 6t^2 - 2}{(t^4 - 2)^2} .
\]

\[
= -\frac{t^6 + 3t^4 + 6t^2 + 2}{(t^4 - 2)^2} .
\]