Dr. Z's Math151 Handout # 2.2 [Limits: A Numerical and Graphical Approach]

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Problem Type 2.2.1 : Given a graph of a function, find the limits from the left, limit from the right, limit (if it exists), at various points, as well as some function values.

Example Problem 2.2.1 : Ex. **7** of section 2.2 in Stewart's book (p. 102).

Steps

Limit from the Left: If the function 'looks' continuous from the left, at the given point, then the limit from the left is the natural 'continuation' (possibly indicated by a hollow dot) from the left.

Limit from the Right: If the function 'looks' continuous from the right, at the given point, then the limit from the right is the natural 'continuation' (possibly indicated by a hollow dot) from the right.

Limit : If BOTH the 'limit from the left' $\lim_{x \to a^-}$ and the 'limit from the right' $\lim_{x \to a^+}$ exist, AND they are equal, then the limit, $\lim_{x \to a}$ exists, and is equal to their common value. Otherwise NOT!

Function Value: To find $f(a)$ from the curve, if the curve passes 'smoothly' through $x = a$, then $f(a)$ has its obvious value. Otherwise it is the value of the filled-in dot.

Example

(a) -1

(b) -2

(c) Does not exist (since the answers to (a) and (b) do not match).

(d) 2

(e) 0

(f) Does not exist (since the answers to (d) and (e) do not match).

(g) 1 (the filled red dot above $x = 2$ is at $(2, 1)$, i.e. where $y = 1$.

(h) 3 (everything is nice and smooth around $(4, 3)$), so all limit exists, and they are all equal to the value of the function at $t = 4$, which is 3.

Problem Type 2.2.2 : Determine the infinite limit

$$\lim_{x \to a^+} \frac{Top(x)}{Bottom(x)} \quad ,$$

where $Bottom(x)$ vanishes at $x = a$ and $Top(x)$ does not.

Example Problem 2.2.2 : Determine the infinite limit

$$\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} \quad .$$

Steps

1. First make sure that indeed the bottom vanishes when you plug in $x = a$, and the top does not. If they both vanish, you may need to use L'Hôpital's rule (coming up later) first.

2. Since we are looking for the limit from the right, plug in the expression a value very close to a, but to its right, for example, $a + .0001$. You will either get something very negative or very positive. If it is very negative, then the answer is $-\infty$, if it very positive, it is ∞.

Example

1. $Top(x) = x - 1$,

$Bottom(x) = x^2(x + 2)$.

$Top(-2) = -3$, $Bottom(-2) = 0$,

so indeed the limit is going to be ∞ or $-\infty$.

2. $Top(-1.9999) = -2.9999$,

$Bottom(-1.9999) = (-1.9999)^2(.0001)$,

so the value of the expression at $x = -1.9999$ is

$(-2.9999)/((-1.9999)^2(.0001))$

which is VERY NEGATIVE (the exact value is not important).

Answer:

$$\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} = -\infty$$