Review Sheet for the Final Exam for Math 151

This review sheet emphasizes the material covered after Exam 2. You should also study the problems in the review sheets for Exam 1 and Exam 2. Your final exam is likely to have problems that do not resemble the review problems.

(1) Evaluate $\int (7x+3)^{10}\,dx$, $\int x^2\sqrt{2x+1}\,dx$, $\int \dfrac{dx}{x(\ln x)^4}$, $\int \dfrac{e^x\,dx}{(e^x+2)^5}$.

(2) Evaluate $\displaystyle\int_0^{3\pi/2} |\cos x|\,dx$, $\displaystyle\int_1^3 \frac{\sqrt{x}-5}{x}\,dx$, $\displaystyle\int_1^2 (\sqrt{x}+3)\left(x-\frac{1}{\sqrt{x}}\right)dx$, $\displaystyle\int_0^1 \frac{dx}{e^x+e^{-x}}$.

(3) Assume f is a continuous function on $[5,7]$ with the property $\displaystyle\int_2^3 f(2x+1)\,dx = 10$. Find $\displaystyle\int_5^7 f(x)\,dx$.

(4) Find $\displaystyle\int_0^2 f(x)\,dx$ when $f(x) = \begin{cases} 2x-1 & \text{if } x \le 1, \\ x^2 & \text{if } x \ge 1. \end{cases}$

(5) Use trig identities to evaluate $\int \cos^2 x + \sin^2 x\,dx$ and $\int \cos^2 x - \sin^2 x\,dx$. Use this to evaluate $\int \cos^2 x\,dx$ and $\int \sin^2 x\,dx$.

(6) Find the left endpoint and midpoint approximations L_4 and M_4 for $y = \frac{1}{x+1}$ on $[0,1]$.

(7) Evaluate $\displaystyle\sum_{k=1}^{n}(2k-1)^2 = 1^2+3^2+5^2+7^2+\cdots+(2n-1)^2$.

(8) Assume f is an even continuous function on $[-1,1]$. Explain why we have the identities

$$\int_{-1}^1 f(x)\,dx = 2\int_0^1 f(x)\,dx \quad \text{and} \quad \int_{-1}^1 x^3 f(x)\,dx = 0 \ .$$

(9) Evaluate $\int \tan^3 x \sec^2 x\,dx$ in two different ways.

(10) Evaluate $\int \sec x\,dx$ using $\sec x = \dfrac{\cos x}{\cos^2 x}$ and $\dfrac{1}{1-u^2} = \dfrac{1/2}{1-u} + \dfrac{1/2}{1+u}$.

(11) Evaluate $\dfrac{d}{dx}\displaystyle\int_0^x \sqrt{1+t^4}\,dt$ and $\dfrac{d}{dx}\displaystyle\int_x^0 \sqrt{1+t^4}\,dt$.

(12) Assume that $s(t)$ is the position of a particle on the x-axis at time t, where distances are measured in feet, and time is measured in seconds. Assume that the particle's acceleration is $\sec^2 t$ feet per second squared at time t. Assume also that the particle's velocity is 3 feet per second at time 0, and $s(0) = 0$. What is the particle's position at time $\pi/4$ seconds?

(13) A bird flies at an altitude of 4 feet and a speed of 3 feet per second from an 8-foot lamppost to a 12-foot lamppost. Find the rate of change with respect to time of the distance between the two shadows of the bird.

(14) Find the maximum and the minimum of $(1 - x^2)^3(10x^2 + 1)$ on $[-1, 1]$.

(15) For the function $f(x) = \dfrac{x^3}{4 + x^2}$, find the inflection points, the intervals where it is concave up and the intervals where it is concave down.

(16) Find the point on the curve $y = \sqrt{\ln x}$, $x > 1$ which is closest to the point $(2, 0)$.

(17) Evaluate $\lim_{x \to 0} \left(\dfrac{\csc x}{x} - \dfrac{\cot x}{x} \right)$.

(18) Find a number a such that the function $f(x) = \begin{cases} x^4 \ln x & \text{if } x > 0 \\ a & \text{if } x = 0 \end{cases}$ is continuous on $[0, \infty)$. Find the minimum of $f(x)$ on $[0, \infty)$ when a is this particular number.

(19) Find the horizontal and vertical asymptotes of $\dfrac{5 - 4x^2 - x^4}{(1 - x^2)(4 - x^2)}$.

(20) Find the horizontal asymptotes of $\dfrac{\sqrt{1 + 9x^2}}{x}$.

(21) Differentiate the following functions:

$$\sin^{-1}(x^3 + 1) \quad , \quad \sqrt{\sec^4 x + x^6} \quad , \quad \frac{x}{\sqrt{x^2 + 1}} \quad , \quad x^2 \tan^{-1}(x^2) \quad , \quad \log_{10}(x^4 + e^x) \ .$$

(22) Suppose that $y = f(x)$ is given implicitly by $x^2y^3 + x^3y = 12$. Assume $f(2) = 1$. Find $f'(2)$.

(23) For the function $f(x) = (3 + x)^2(x^2 - 1)$, find the local maxima, the local minima, the intervals where it is increasing, the intervals where is is decreasing, the inflections points, the intervals where it is concave up, and the intervals where it is concave down.

(24) Assume that $f(x)$ is a differentiable function such that $f(2) = 3$ and $f'(x) \leq -4$ for all x. What can we say about $f(5)$?