Dr. Z’s Math151 Handout #5.4 [Indefinite Integrals and the Net Change Theorem]

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Problem Type 5.4.1: Find the general indefinite integral
\[ \int f(Var) dVar \ . \]

Example Problem 5.4.1: Find
\[ \int (1-t)(2+t^2) dt \ . \]

**Steps**

1. Use algebra and/or trig. to simplify the integrand \( f(Var) \) as much as possible.

   \[ (1-t)(2+t^2) = 2 - 2t + t^2 - t^3, \]  
   \[ \int (1-t)(2+t^2) dt = \int (2-2t+t^2-t^3) dt \ . \]

2. Use the addition rule of integration to split the integral into its constituent parts, and then find an antiderivative for each piece using the table in your head (\( \int t^n dt = \frac{t^{n+1}}{n+1} \), etc.). Do not bother with the \( C \) in the intermediate steps.

   \[ \int (2-2t+t^2-t^3) dt = 2 \int 1 dt - 2 \int t dt + \int t^2 dt - \int t^3 dt \ . \]
   \[ = 2t - 2(t^2/2) + t^3/3 - t^4/4 \ . \]

3. Add \(+C\) to the answer. \( C \) stands for arbitrary constant.

   \[ \text{Ans.} = 2t - t^2 + \frac{t^3}{3} - \frac{t^4}{4} + C \ . \]
Problem Type 5.4.2: Evaluate the (definite) integral

\[ \int_{a}^{b} f(\text{Var})d\text{Var} \]

Example Problem 5.4.2: Evaluate the (definite) integral

\[ \int_{-1}^{1} (u^5 - u^3 + u^2)\,du \]

Steps

1. First find the indefinite integral, like we did in 5.4.1, and get an expression in \text{Var}, except that you need not bother with the \( C \).

\[ \int f(u)\,du = F(u) \]

2. Now stick the limit of integrations on the left, and an evaluation line on the right with the corresponding limits.

\[ \int_{a}^{b} f(u)\,du = F(u)\bigg|_{a}^{b} \]

3. Compute \( F(b) - F(a) \) by plugging-in the upper limit (\( b \)) and subtracting from it \( F \) plugged-in the lower limit (\( a \)).

\[ \left( \frac{(1)^6}{6} - \frac{(1)^4}{4} + \frac{(1)^3}{3} \right) - \left[ \frac{(-1)^6}{6} - \frac{(-1)^4}{4} + \frac{(-1)^3}{3} \right] = \frac{2}{3}. \]