Dr. Z's Math151 Handout #3.4 [Derivatives of Trigonometric Functions]

By Doron Zeilberger

Problem Type 3.4.1 : Differentiate an expression involving products and/or quotients of expressions containing *trig* functions.

Example Problem 3.4.1: Differentiate

$$y = \frac{1+\cos x}{x+\sin x} \quad .$$

Steps

1. Use the product or quotient rule to differentiate the given expression, remembering that $(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$. You don't really need to remember the more obscure formulas $(\csc x)' = -\csc x \cot x$, $(\sec x)' = \sec c \tan x$, $(\cot x)' = -\csc^2 x$, since you can always replace $\csc x$ by $1/\sin x$, etc.

2. Use algebra, and possibly trig identities, the most important one being

$$\sin^2 + \cos^2 = 1 \quad ,$$

to simplify.

Example

1.

$$(\frac{1+\cos x}{x+\sin x})' =$$

$$\frac{(1+\cos x)'(x+\sin x) - (1+\cos x)(x+\sin x)'}{(x+\sin x)^2} =$$

$$\frac{(-\sin x)(x+\sin x) - (1+\cos x)(1+\cos x)}{(x+\sin x)^2} \quad .$$

2.

$$\frac{(-x\sin x - \sin^2 x) - (1 + 2\cos x + \cos^2 x)}{(x+\sin x)^2} =$$

$$\frac{-1 - x\sin x - 2\cos x - (\sin^2 x + \cos^2 x)}{(x+\sin x)^2} =$$

$$\frac{-1 - x\sin x - 2\cos x - (1)}{(x+\sin x)^2} =$$

$$\frac{-2 - x\sin x - 2\cos x}{(x+\sin x)^2} \quad .$$

Ans.: $\frac{-2-x\sin x-2\cos x}{(x+\sin x)^2}$.

Problem Type 3.4.2 : Find limits of expressions involving trig. functions. (usually at 0).

Example Problem 3.4.2: Find the limit

$$\lim_{x \to 0} \frac{\sin 7x}{\sin 5x}$$

Steps

1. Important Fact: When you are taking a limit as $x \to 0$, and the argument of sin is 0 when $x = 0$, then you can 'forget about the sin', meaning replacing $\sin(WhatEver)$ by $WhatEver$.

Example

1.

$$\lim_{x \to 0} \frac{\sin 7x}{\sin 5x} =$$

$$\lim_{x \to 0} \frac{7x}{5x} =$$

$$\lim_{x \to 0} \frac{7}{5} =$$

$$\frac{7}{5} \quad .$$