Dr. Z's Math151 Handout #2.8 [Derivatives]

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Problem Type 2.8.1 : Find $f'(a)$ if $f(variable) = Expression(variable)$.

Example Problem 2.8.1: Find $f'(a)$ if

$$f(s) = \frac{s+3}{2s+1}$$

Steps

1. This is exactly like Problem 4.1 (section 2.7), but you have to know that $f'(a)$ **means** *slope* at $x = a$. So

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad .$$

You think of a as something *fixed*, since the variable is x. Of course if another variable is used instead of x (s in this example, then you use s instead of x.

Example

1.

$$f'(a) = \lim_{s \to a} \frac{f(s) - f(a)}{s - a} = \lim_{s \to a} \frac{\frac{s+3}{2s+1} - \frac{a+3}{2a+1}}{s - a} =$$

$$\lim_{s \to a} \frac{\frac{(s+3)(2a+1)-(2s+1)(a+3)}{(2s+1)(2a+1)}}{s - a} =$$

$$\lim_{s \to a} \frac{(s+3)(2a+1) - (2s+1)(a+3)}{(2s+1)(2a+1)(s-a)} =$$

$$\lim_{s \to a} \frac{(s+3)(2a+1) - (2s+1)(a+3)}{(2s+1)(2a+1)(s-a)} =$$

$$\lim_{s \to a} \frac{2as + s + 6a + 3 - 2sa - 6s - a - 3}{(2s+1)(2a+1)(s-a)} =$$

$$\lim_{s \to a} \frac{5a - 5s}{(2s+1)(2a+1)(s-a)} =$$

$$\lim_{s \to a} \frac{-5(s-a)}{(2s+1)(2a+1)(s-a)} = \lim_{s \to a} \frac{-5}{(2s+1)(2a+1)} =$$

$$\frac{-5}{(2s+1)(2a+1)} \Big|_{s=a}$$

$$\frac{-5}{(2a+1)(2a+1)} = \frac{-5}{(2a+1)^2}$$