Dr. Z's Math151 Handout #2.7 [Tangents, Velocities, and Other Rates of Change]

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Problem Type 2.7.1 : Find an equation for the tangent line to the curve at the given point. $y = f(a)$, $(a, f(a))$.

Example Problem 2.7.1: Find an equation for the tangent line to the curve at the given point. $y = x^2 + 2x$, $(1, 3)$.

Steps

1. Very soon we will do this using derivatives and differentiation rules. In this section you are supposed to compute the slope of the tangent at the given point from first-principle, using

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \quad .$$

2. Having found the slope m, the equation of the line is $(y - f(a)) = m(x - a)$.

Example

1. Here $a = 1$ and $f(x) = x^2 + 2x$. The slope is:

$$m = \lim_{x \to 1} \frac{x^2 + 2x - (1^2 + 2 \cdot 1)}{x - 1} = \lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x+3)(x-1)}{(x-1)} = \lim_{x \to 1} \frac{(x+3)}{1} = 1+3 = 4 \quad .$$

2. $(y - 3) = 4(x - 1)$, and bringing y alone on the left, we get the standard (slope-intercept form) form $y = 4x - 1$.

Ans.: The equation of the tangent to the curve $y = x^2 + 2x$ at the point $(1, 3)$ is $y = 4x - 1$.

Problem Type 2.7.2 : You are given a plot of distance (s) versus time (t), by looking at the slope, you have to answer questions about the velocity at various points.

Example Problem 2.7.2: ex. 15 of sect. 2.7 (p. 156).

Steps	**Example**
a. The initial velocity is the slope at $t = 0$ (the origin).	**a.** The tangent is horizontal (along the t-axis), hence the slope alias velocity is 0. **Ans.**: 0.
b. You look which point has a steeper slope.	**b.** The slope of the tangnet at C is steeper than the slope of the tangnet at B. Hence the car was going faster at C.
c. If the slopes are getting steeper and steeper (convex up) near a given point in time, than it is speeding up, otherwise it is slowing down. If a segment is a straight line, then it is going at a constant speed.	**c.** A: speeding up. B: slowing down. C: speeding up.
d. The speed in a segment that is a straight line is constant and equals the slope.	**d.** From D to E it is a *horizontal* straight line, hence the constant speed is 0, and the car does not move at all.