Dr. Z's Math151 Handout # 2.3 [Calculating Limits Using the Limit Laws]

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Problem Type 2.3.1: Evaluate the limit if it exists:

$$\lim_{x \to a} f(x)$$

Example Problem 2.3.1 : Evaluate the limit if it exists:

$$\lim_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4}$$

Steps

1. Try to plug $x = a$ into the function, if it makes sense (i.e. the denominator is not zero), then the limit is that value. For example, $\lim_{x \to 2}(x^2-1)/(x+1) = (2^2-1)/(2+1) = 1$. If the top is non-zero and the bottom 0, then it does not exists. If you get $0/0$ then **SIMPLIFY AS MUCH AS POSSIBLE**.

2. Try to plug in $x = a$ again, if you still get $0/0$, go back to step **1**. Otherwise you are done.

Example

1. Plugging in $x = -4$ in $\frac{x^2+5x+4}{x^2+3x-4}$ gives $\frac{(-4)^2+5(-4)+4}{(-4)^2+3(-4)-4}$, which is $0/0$. So we must **simplify**. Factoring the top and bottom we get

$$\frac{x^2+5x+4}{x^2+3x-4} = \frac{(x+1)(x+4)}{(x-1)(x+4)} = \frac{x+1}{x-1}$$

2. Plugging in $x = -4$ in

$$\frac{x+1}{x-1}$$

gives

$$\frac{-4+1}{-4-1} = \frac{-3}{-5} = \frac{3}{5}, \quad .$$

Final Answer: $3/5$.

Problem Type 2.1.2:

Use the **Squeeze Theorem** to prove that

$$\lim_{x\to a} GoesToZero(x) \cdot Bounded(x) = 0$$

Example Problem 2.1.2 : Use the **Squeeze Theorem** to prove that

$$\lim_{x\to 0} \sqrt{x^3 + x^2}\,\sin(\pi/x) = 0$$

Steps	**Example**
1. Show that $$\lim_{x\to 0} GoesToZero(x)$$ indeed equals 0.	**1.** $$\lim_{x\to 0} \sqrt{x^3+x^2} = \sqrt{0^3+0^2} = \sqrt{0} = 0.$$
2. Show that $Bounded(x)$ is indeed bounded, at least near $x = a$.	**2.** The sin function is always between -1 and 1 hence is always bounded. $\square$