# 2016 Robbins Prize

The 2016 David P. Robbins Prize is awarded to Christoph Koutschan, Manuel Kauers, and Doron Zeilberger for their paper, "Proof of George Andrews's and David Robbins's $q$-TSPP conjecture," Proc. Natl. Acad. Sci. USA 108 (2011), 2196–2199.

A plane partition $\pi$ may be thought of as a finite set of triples of natural numbers with the property that if $(i, j, k)$ is in $\pi$, then so is every $(i', j', k')$ with $i' \leq i$, $j' \leq j$ and $k' \leq k$. The symmetric group $S_3$ acts on plane partitions by permuting the coordinate axes. A plane partition $\pi$ may also be "complemented" by constructing the set of points not in $\pi$ that are in the smallest box enclosing $\pi$, and then reflecting them through the center of the box. The set of plane partitions that is invariant under some subgroup of these transformations is known as a "symmetry class of plane partitions."

In the early 1980's, the combined experimental observations of several people, including David Robbins himself, suggested that *all* the symmetry classes of plane partitions were enumerated by a family of remarkably simple product formulas. In some cases, this was also found to be true of the $q$-analogs of these formulas—generating functions where $q$ tracks some parameter of interest. Simple combinatorial formulas usually have simple combinatorial explanations, but surprisingly, that is not the case here. Many of the formulas are exceedingly difficult to prove, and to this day there is no unified proof that explains why they are all so simple and so similar. When simple combinatorial formulas fail to have simple combinatorial proofs, it usually means that something deep is going on beneath the surface. Indeed, the study of symmetry classes of plane partitions has revealed unexpected connections between several areas, including statistical mechanics, the representation theory of quantum groups, alternating sign matrices, and lozenge tilings.

By 1995, all the formulas had been proved but one: the $q$-TSPP (totally symmetric plane partitions, where $q$ tracks the number of orbits of triples) formula, independently conjectured by George Andrews and David Robbins. The paper by Kauers, Koutschan, and Zeilberger, along with supporting computer files on Koutschan's website, finally established the correctness of this last conjectured formula. It is a tour de force of experimental mathematics.

The authors' starting point was a 1989 paper of Okada that reduced the problem to a conjecture on the values of the determinants of a special sequence of matrices. The form of the matrices enables one to verify the conjecture by Zeilberger's "holonomic ansatz", in which one hypothesizes a system of auxiliary functions that would certify the correctness of the determinant evaluation. If these auxiliary functions exist, then one can search for them computationally. Once found, the functions must be shown to satisfy certain recurrences in order to complete the proof. The authors performed the search, found the candidate auxiliary functions, and empirically verified that they seemed to satisfy the necessary recurrences. Although techniques existed that might in principle find proofs for the recurrences, the necessary computations seemed far beyond the reach of current computers, and the authors initially announced a semi-rigorous but not fully rigorous proof. However, the authors then developed "creative telescoping" techniques that dramatically simplified the necessary computations, resulting in a fully rigorous proof of the $q$-TSPP formula.

The Robbins Prize is awarded to a novel research paper in algebra, combinatorics, or discrete mathematics with a significant experimental component. The $q$-TSPP paper is a shining example. The conjecture itself was born from experimental observations, and the proof involved the development of new experimental mathematical techniques that are sure to solve many problems.

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