Solutions to Dr. Z.'s Math 354 REAL Quiz #5

1. (8 pts.) Use the **simplex method** (no credit for other methods) to solve the following linear programming problem.

Maximize $z = x + y$, subject to

$$2x + y \leq 6 \quad , \quad x + 2y \leq 6 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad .$$

Sol. to 1: We first convert to **canonical form** by introducing the **slack variables** u, v.

The problem now is

Maximize $z = x + y$ subject to the restrictions

$$2x + y + u = 6 \quad , \quad x + 2y + v = 6 \quad ,$$

$$x \geq 0 \quad , \quad y \geq 0 \quad , \quad u \geq 0 \quad , \quad v \geq 0 \quad .$$

Writing $z = x + y$ as $-x - y + z = 0$, the *initial tableau* is

	x	y	u	v	z	
u	2	1	1	0	0	6
v	1	2	0	1	0	6
	-1	-1	0	0	1	0

We look at the **most negative** entry in the bottom row. In this case there is a *tie*, so we pick any of them. Let's pick the first column as the **pivot** column. To decide on the **pivot row**, we form the θ-ratios. The θ-ratio of the first row is $6/2 = 3$, while that of the second row is $6/1 = 6$. We pick the row that gives the smallest, so the **pivot row** is the first row, and the **pivot entry** is the $(1,1)$ entry of the tableau. Also the **entering** basic-variable is x and the **departing** basic-variable is u. Let's make the pivot entry 1 by dividing the first row by 2, doing $\frac{1}{2}r_1 \rightarrow r_1$, and indicate by arrows the entering and departing basic variables.

	$x^{\downarrow}$	y	u	v	z	
$\leftarrow u$	1	$1/2$	$1/2$	0	0	3
v	1	2	0	1	0	6
	-1	-1	0	0	1	0

Since x is going to be the new basic variable, we must make everything in the first column (except for the pivot) 0. We perform the elementary row operations $r_2 - r_1 \to r_2$ and $r_3 + r_1 \to r_3$ and at the same time replace the u on the basic variable column by x.

$$
\begin{array}{c|ccccc|c}
 & x & y & u & v & z & \\
 & & & & & & \\
x & 1 & 1/2 & 1/2 & 0 & 0 & 3 \\
v & 0 & 3/2 & -1/2 & 1 & 0 & 3 \\
 & & & & & & \\
 & 0 & -1/2 & 1/2 & 0 & 1 & 3
\end{array} .
$$

Now the most negative entry in the bottom row is the $-1/2$ (in fact it is the only one) at the second column. So the second column is the **pivot column**. The θ-ratio of the first row is $3/(1/2) = 6$, while that if the second row is $3/(3/2) = 2$. Since the second is the smallest, the **pivot row** is the second row and the **pivot entry** is the $(2, 2)$-entry. Let's first make it 1 by doing $\frac{2}{3}r_2 \to r_2$, and indicate by arrows the entering basic variable (y), and the departing basic variable (v) .

We get the tableau

$$
\begin{array}{c|ccccc|c}
 & x & y^{\downarrow} & u & v & z & \\
 & & & & & & \\
x & 1 & 1/2 & 1/2 & 0 & 0 & 3 \\
\leftarrow v & 0 & 1 & -1/3 & 2/3 & 0 & 2 \\
 & & & & & & \\
 & 0 & -1/2 & 1/2 & 0 & 1 & 3
\end{array} .
$$

Everything in the pivot column, i.e. the y-column, except for the pivot entry must be 0.

Doing $r_1 - \frac{1}{2}r_2 \to r_1$ and $r_3 + \frac{1}{2}r_2 \to r_3$ gives the tableau

$$
\begin{array}{c|ccccc|c}
 & x & y & u & v & z & \\
 & & & & & & \\
x & 1 & 0 & 2/3 & -1/3 & 0 & 2 \\
y & 0 & 1 & -1/3 & 2/3 & 0 & 2 \\
 & & & & & & \\
 & 0 & 0 & 1/3 & 1/3 & 1 & 4
\end{array} .
$$

At long last, there are no negative entries in the bottom row, so we are done.

The basic variables are now x and y and we have $x = 2$ and $y = 2$. The non-basic variables, u and v, are of course 0. So the optimal solution is $(x, y, u, v) = (2, 2, 0, 0)$, and the optimal value is 4.

Going back to the original problem, we can ignore the slack variable, and get.

Ans. to 1: The optimal solution is $x = 2$, $y = 2$. The optimal value is $z = 4$.