Solutions to Dr. Z.'s Math 354 REAL Quiz #2

1. (8 pts.) Formulate the following problem as a linear programming problem. State all the constraints and state the goal function.

A certain computer maker has three factories. In City 1, City 2, and City 3. it also has two stores, in City 1', and City 2'.

- The factory in City 1 produces 2500 computers per week.
- The factory in City 2 produces 2000 computers per week.
- The factory in City 3 produces 1500 computers per week.
- The store in City 1' sells 4000 computers per week.
- The store in City 2' sells 2000 computers per week.
- The distances from City 1' to City 1, City 2, City 3 are 1000, 1500, and 1200 miles respectively
- The distances from City 2' to City 1, City 2, City 3 are 1400, 1100, and 900 miles respectively

Assume that the cost of transportation is proportional to the distance. Let x_{ij} be the number of computers shipped from City i to City j' ($i = 1, 2, 3$, $j = 1, 2$). You want to decide how many computers to ship from each factory to each store so as to minimize the transportation cost.

Sol. to 1: **Version of Feb. 8, 2019 (more details)**

Congratulations to the people who got it completely right (about %40). About %20 were on the right track (and possibly ran out of time), but about %40 were completely clueless. PLEASE come to free tutoring and/or office hours, also from the very good TA at-large, Mr. Matthew Hohertz, to make sure that you **understand** how to set it up. It requires a certain *mathematical maturity*, but once you understand how to do it, it should not be too hard.

Step 1 (Most important!): Decide on the **variables**! If you look at the end of the problem it asks

"..how many computers to ship from each factory to each store so as to minimize the transportation cost"

Since there are 3 factories and 2 stores, and for each pair $(factory, store)$, you have to decide on the number of computers to ship from the factory to the store, there are $3 \cdot 2 = 6$ variables!

- How many computers to ship from factory 1 to store $1'$: x_{11}
- How many computers to ship from factory 1 to store $2'$: x_{12}
- How many computers to ship from factory 2 to store $1'$: x_{21}

- How many computers to ship from factory 2 to store $2'$: x_{22}
- How many computers to ship from factory 3 to store $1'$: x_{31}
- How many computers to ship from factory 3 to store $2'$: x_{32}

Hence the set of **variables** is $x_{11}, x_{12}, x_{21}, x_{22}, x_{31}, x_{32}$.

Comment: Quite a few people got this part wrong, and they had three variables. Once you get this part wrong every thing after that is **meaningless**.

Since the cost of shipping is **proportional** to the distance, if the distance from City i' to City j is c_{ij} then the cost of shipping x_{ij} computers is $c_{ij}x_{ij}$.

According to the data

$$c_{11} = 1000 \quad , \quad c_{21} = 1500 \quad , \quad c_{31} = 1200 \quad ,$$

$$c_{12} = 1400 \quad , \quad c_{22} = 1100 \quad , \quad c_{32} = 900 \quad .$$

Adding up all the $c_{ij}x_{ij}$ gives that the **goal function**, that we have to minimize, is

$$z = 1000x_{11} + 1500x_{21} + 1200x_{31} + 1400x_{12} + 1100x_{22} + 900x_{32} \quad .$$

Now we have to worry about the constraints.

There are two kinds of constraints.

Supply Side constraints

The number of computers shipped from Factory 1 is $x_{11} + x_{12}$ (since there are two stores, x_{11} of them go store 1 x_{12} of them go store 2, so the total number of computers shipped from Factory 1 is the sum). Ideally you would like to get rid of all of them, but you don't know that beforehand. Since you can't ship more computers than you have, the constraint is

$$x_{11} + x_{12} \leq 2500.$$

Similarly, for factory in city 2

$$x_{21} + x_{22} \leq 2000.$$

and for factory in city 3

$$x_{31} + x_{32} \leq 1500.$$

Demand Side constraints

The number of computers **received** at store 1 is $x_{11} + x_{21} + x_{31}$, since x_{11} came from factory 1, x_{21} came from factory 2, and x_{31} came from factory 3. Ideally you would like to sell all of them, but you don't know that beforehand. At any rate, in order that you will not get in trouble, you need to have **at least** 4000. So we have the constraint

$$x_{11} + x_{21} + x_{31} \geq 4000 \quad .$$

Using the same reasoning for store 2, we have

$$x_{12} + x_{22} + x_{32} \geq 2000$$

Of course don't forget that you can't ship a negative number of computers. So we have the six simple constraints

$$x_{11} \geq 0 \quad , \quad x_{21} \geq 0 \quad , \quad x_{31} \geq 0 \quad , \quad x_{21} \geq 0 \quad , \quad x_{22} \geq 0 \quad , \quad x_{32} \geq 0 \quad .$$

Finally, we have

Ans. to 1:

Minimize

$$z = 1000x_{11} + 1500x_{21} + 1200x_{31} + 1400x_{12} + 1100x_{22} + 900x_{32} \quad .$$

subject to the restrictions

$$x_{11} + x_{12} \leq 2500 \quad , \quad x_{21} + x_{22} \leq 2000 \quad , \quad x_{31} + x_{32} \leq 1500 \quad ,$$

$$x_{11} + x_{21} + x_{31} \geq 4000 \quad , \quad x_{12} + x_{22} + x_{32} \geq 2000 \quad ,$$

$$x_{11} \geq 0 \quad , \quad x_{21} \geq 0 \quad , \quad x_{31} \geq 0 \quad , \quad x_{12} \geq 0 \quad , \quad x_{22} \geq 0 \quad , \quad x_{32} \geq 0 \quad .$$