Solutions to Dr. Z.'s Math 354 REAL Quiz # 1

1. (8 pts.) By using **Gauss-Jordan Reduction** (no credit for other methods), find all solutions to the following linear system:

$$x + 2y + z = 4 \quad ,$$

$$2x + y + 3z = 6 \quad ,$$

$$4x + 3y + z = 8 \quad .$$

Sol. of 1: The **augmented matrix** is

$$\begin{bmatrix} 1 & 2 & 1 & | & 4 \\ 2 & 1 & 3 & | & 6 \\ 4 & 3 & 1 & | & 8 \end{bmatrix} \quad .$$

Doing $r_2 - 2r_1 \to r_2$ and $r_3 - 4r_1 \to r_3$ gives

$$\begin{bmatrix} 1 & 2 & 1 & | & 4 \\ 0 & -3 & 1 & | & -2 \\ 0 & -5 & -3 & | & -8 \end{bmatrix} \quad .$$

Doing $-\frac{1}{3}r_2 \to r_2$ and $-\frac{1}{5}r_3 \to r_3$ gives

$$\begin{bmatrix} 1 & 2 & 1 & | & 4 \\ 0 & 1 & -\frac{1}{3} & | & \frac{2}{3} \\ 0 & 1 & \frac{3}{5} & | & \frac{8}{5} \end{bmatrix} \quad .$$

Doing $r_3 - r_2 \to r_3$ gives

$$\begin{bmatrix} 1 & 2 & 1 & | & 4 \\ 0 & 1 & -\frac{1}{3} & | & \frac{2}{3} \\ 0 & 0 & \frac{14}{15} & | & \frac{14}{15} \end{bmatrix} \quad .$$

Doing $\frac{15}{14}r_3 \to r_3$ gives

$$\begin{bmatrix} 1 & 2 & 1 & | & 4 \\ 0 & 1 & -\frac{1}{3} & | & \frac{2}{3} \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \quad .$$

Now it is in **row-echelon form**.

Doing $r_1 - r_3 \to r_1$ and $r_2 + \frac{1}{3}r_3 \to r_2$ gives

$$\begin{bmatrix} 1 & 2 & 0 & | & 3 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \quad .$$

Doing $r_1 - 2r_2 \rightarrow$ gives

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \quad .$$

Now it is in **reduced row-echelon form**. In everyday notation this is

$$x = 1 \quad , \quad y = 1 \quad , \quad z = 1 \quad .$$

Ans. to 1.: The only solution of the system is $x = 1, y = 1, z = 1$ and in vector notation

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad .$$