Solutions to Attendance Quiz for Lecture 4

1. Write the following linear programming problem in matrix form.

Maximize $z = x + 3y$ subject to the restrictions

$$5x + y \leq 5 \quad , \quad -x + 4y \leq 8 \quad , \quad -11x + 4y \leq 12 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad .$$

Sol. to 1:

$$Maximize \quad z = \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad ,$$

subject to

$$\begin{bmatrix} 5 & 1 \\ -1 & 4 \\ -11 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \leq \begin{bmatrix} 5 \\ 8 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \geq \mathbf{0} \quad .$$

2. By introducing slack variables (give them names), write the above problem in **canonical form** (expressed, again, in matrix form).

Sol. to 2.: Introducting **slack variables** u, v, and w, the canonical form of the same problem (in scalar notation) is

Maximize $z = x + 3y$ subject to the restrictions

$$5x + y + u = 5 \quad , \quad -x + 4y + v = 8 \quad , \quad -11x + 4y + w = 12 \quad ,$$

$$x \geq 0 \quad , \quad y \geq 0 \quad , \quad u \geq 0 \quad , \quad v \geq 0 \quad , \quad w \geq 0 \quad .$$

In **matrix form**, this becomes

$$Maximize \quad z = \begin{bmatrix} 1 & 3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ u \\ v \\ w \end{bmatrix} \quad ,$$

subject to

$$\begin{bmatrix} 5 & 1 & 1 & 0 & 0 \\ -1 & 4 & 0 & 1 & 0 \\ -11 & 4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 12 \end{bmatrix} \quad ,$$

$$\begin{bmatrix} x \\ y \\ u \\ v \\ w \end{bmatrix} \geq \mathbf{0} \quad .$$