Solutions to Attendance Quiz for Lecture 2

1. Is the following *subset* of R^4 also a *subspace* of R^4?

$$\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in R^4 \,|\, x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0 \} \quad .$$

Explain!

Solution to 1. No. The axiom that if $\mathbf{v} \in V$ and $k \in R$ then $k\mathbf{v} \in V$ is violated. For example if $\mathbf{v} = [1, 0, 0, 0]^T$ and $k = -1$, then $k\mathbf{v} = [-1, 0, 0, 0]^T$ and does not belong to $\mathbf{V}$, since if it were, then x_1, would have to be -1, and that's against the defining rule of V.

2. Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 6 & 8 \end{bmatrix} \quad .$$

Sol. of 2

First way: We need to bring the matrix to **row-echelon form** (we do **not** need to go all the way to reduced-row echelon form, that's a waist of time).

Doing $r_2 - 2r_1 \rightarrow r_2$ and $r_3 - 3r_1 \rightarrow r_3$ gives the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \\ 0 & 0 & -1 \end{bmatrix} \quad .$$

Doing $r_3 - r_2 \rightarrow r_3$ gives

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad .$$

Doing $-r_2 \rightarrow r_2$ gives

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad .$$

Now it is in row-echelon form. The number of pivots (alias the number of not all-zero rows) is 2 hence the rank is 2.

Ans. to 2: The rank is 2.

Second way: by **inspection** the third row is the sum of the first and second rows, hence is a **linear combination** of them, hence can be kicked out. So it is the same as finding the rank of

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix} \quad .$$

Again by inspection the second row is **not** a constant multiple of the first (or vice versa) hence they are **linearly independent**. Hence the dimension of the row-space is 2, hence the rank is 2.

Ans. to 2: The rank is 2.