Solutions to Attendance Quiz for Lecture 15

1. Find the initial basic feasible solution using (a) the minimum cost rule (b) Vogel's method

$$\mathbf{C} = \begin{bmatrix} 5 & 2 & 3 & 6 \\ 2 & 7 & 7 & 4 \\ 1 & 3 & 6 & 9 \end{bmatrix} \quad , \quad \mathbf{s} = \begin{bmatrix} 100 \\ 80 \\ 140 \end{bmatrix} \quad , \quad \mathbf{d} = \begin{bmatrix} 60 \\ 60 \\ 80 \\ 120 \end{bmatrix} \quad .$$

Sol. to 1(a): At the very beginning there is nothing assigned, and we have

$$\begin{bmatrix} 0 & 0 & 0 & 0 & | & \mathbf{100} \\ 0 & 0 & 0 & 0 & | & \mathbf{80} \\ 0 & 0 & 0 & 0 & | & \mathbf{140} \\ & & & & | & \\ \mathbf{60} & \mathbf{60} & \mathbf{80} & \mathbf{120} & & \end{bmatrix}$$

We fill-in row by row.

• The cheapest cell in row 1 is $[1, 2]$, so we assign as much as possible to it, since the current demand in column 2 is 60, it follows that the most that we can do is to assign 60 to cell $[1, 2]$. This leaves 40 units to allocate to row 1.

The second cheapest cell in row 1 is $[1, 3]$, and since the current demand in column 3 is 80, we can assign the remaining 40 to that cell (without causing an overload). The current, (still partial) feasible solution is

$$\begin{bmatrix} 0 & 60 & 40 & 0 & | & \mathbf{0} \\ 0 & 0 & 0 & 0 & | & \mathbf{80} \\ 0 & 0 & 0 & 0 & | & \mathbf{140} \\ & & & & | & \\ \mathbf{60} & \mathbf{0} & \mathbf{40} & \mathbf{120} & & \end{bmatrix}$$

• The cheapest cell in row 2 is $[2, 1]$. Ideally, we would like to assign all the available supply of row 2 , namely 80, to it. Alas, the maximum it can accommodate is 60 (since that's the current demand in column 1. The remaining 20 go to the second cheapest cell in row 2, namely cell $[2, 4]$, and since the current demand of column 4 is 120 it can accomodate it. The current partial feasible solution is

$$\begin{bmatrix} 0 & 60 & 40 & 0 & | & \mathbf{0} \\ 60 & 0 & 0 & 20 & | & \mathbf{0} \\ 0 & 0 & 0 & 0 & | & \mathbf{140} \\ & & & & | & \\ \mathbf{0} & \mathbf{0} & \mathbf{40} & \mathbf{100} & & \end{bmatrix}$$

• The cheapest cell in row 3 is $[3, 1]$. Alas, column 1 is all booked-up, so we try the second cheapest cell, $[3, 2]$. But column 2 is also booked-up (the remaining demand is also 0). We go reluctantly to the next cheapest cell, $[3,3]$ and since the current demand for column 3 is 40 we allocate 40 to cell $[3,3]$, and the remaining 100 go to cell $[3,4]$. What we have now is:

$$\begin{bmatrix} 0 & 60 & 40 & 0 & | & \mathbf{0} \\ 60 & 0 & 0 & 20 & | & \mathbf{0} \\ 0 & 0 & 40 & 100 & | & \mathbf{0} \\ & & & & | & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & & \end{bmatrix}$$

Ans. to 1(a): The initial basic feasible solution, using the Minimal Cost Rule is

$$\begin{bmatrix} 0 & 60 & 40 & 0 \\ 60 & 0 & 0 & 20 \\ 0 & 0 & 40 & 100 \end{bmatrix} \quad .$$

The **initial cost** is

$$60 \cdot 2 + 40 \cdot 3 + 60 \cdot 2 + 20 \cdot 4 + 40 \cdot 6 + 100 \cdot 9 = 1580 \quad .$$

Sol. to 1(b): We have to also modify the cost matrix. Since I don't like to cross-out, I will indicate by 'underline' entries in the cost matrix that no longer should be considered. We also indicate in **boldface** for each row and each column the difference between the cost of the cheapest cell and the second-cheapest cell.

At the very beginning there is nothing assigned, and we have

$$\begin{bmatrix} 0 & 0 & 0 & 0 & | & \mathbf{100} \\ 0 & 0 & 0 & 0 & | & \mathbf{80} \\ 0 & 0 & 0 & 0 & | & \mathbf{140} \\ & & & & | & \\ \mathbf{60} & \mathbf{60} & \mathbf{80} & \mathbf{120} & & \end{bmatrix} \quad , \quad \mathbf{C} = \begin{bmatrix} 5 & 2 & 3 & 6 & |\mathbf{1} \\ 2 & 7 & 7 & 4 & |\mathbf{2} \\ 1 & 3 & 6 & 9 & |\mathbf{2} \\ & & & & \\ \mathbf{1} & \mathbf{1} & \mathbf{3} & \mathbf{2} & \end{bmatrix} \quad ,$$

The largest difference is at the **third column**. The **cheapest** cell of the third column is the cell $[1,3]$, the current available supply in the first row is 100, and in the third column is 80. Since 80 is the smallest, we assign 80 to cell $[1,3]$, update the supply of row 1 to $100-80$, update the demand of column 3 to 0, and remove all the cells of the third column of the cost matrix from future consideration (since the "budget" of the third column just became 0)

Right now we have (we also indicate for each surviving line of the cost matrix the difference between the cheapest and second-cheapest cell):

$$\begin{bmatrix} 0 & 0 & 80 & 0 & | & \mathbf{20} \\ 0 & 0 & 0 & 0 & | & \mathbf{80} \\ 0 & 0 & 0 & 0 & | & \mathbf{140} \\ & & & & | & \\ \mathbf{60} & \mathbf{60} & \mathbf{0} & \mathbf{120} & & \end{bmatrix} \quad , \quad \mathbf{C} = \begin{bmatrix} 5 & 2 & \underline{3} & 6 & |\mathbf{3} \\ 2 & 7 & \underline{7} & 4 & |\mathbf{2} \\ 1 & 3 & \underline{6} & 9 & |\mathbf{2} \\ & & & & \\ \mathbf{1} & \mathbf{1} & \mathbf{na} & \mathbf{2} & \end{bmatrix} \quad ,$$

The largest difference is at the **first row** . The **cheapest** cell of the first row is the cell $[1,2]$, the current available supply in the first row is 20, and in the second column is 60. Since 20 is the smallest, we assign 20 to cell $[1,2]$, update the supply of row 1 to $20-20=0$, update the demand of column 2 to $60-20=40$, and remove all the cells of the first row of the cost matrix from future consideration (since the "budget" of the first row just became 0). Right now we have (we also indicate for each surviving line of the cost matrix the difference between the cheapest and second-cheapest cell.

$$
\left[\begin{array}{cccc|c}
0 & 20 & 80 & 0 & \mathbf{0} \\
0 & 0 & 0 & 0 & \mathbf{80} \\
0 & 0 & 0 & 0 & \mathbf{140} \\
 & & & & \\
\mathbf{60} & \mathbf{40} & \mathbf{0} & \mathbf{120} &
\end{array}\right] \quad , \quad \mathbf{C} = \left[\begin{array}{cccc|c}
\underline{5} & \underline{2} & \underline{3} & \underline{6} & \mathbf{na} \\
2 & 7 & \underline{7} & 4 & \mathbf{2} \\
1 & 3 & \underline{6} & 9 & \mathbf{2} \\
 & & & & \\
\mathbf{1} & \mathbf{4} & \mathbf{na} & \mathbf{5} &
\end{array}\right] \quad ,
$$

The largest difference is at the **fourth column** . The **cheapest** (surviving) cell of the first column is the cell $[2,4]$, the current available supply in the second row is 80, and in the fourth column is 120. Since 80 is the smallest, we assign 80 to cell $[2,4]$, update the supply of row 2 to $80-80=0$, update the demand of column 4 to $120-80=40$, and remove all the cells of the second row of the cost matrix from future consideration (since the budget of the second row just became 0). Right now we have (we also indicate for each surviving line of the cost matrix the difference between the cheapest and second-cheapest cell.

$$
\left[\begin{array}{cccc|c}
0 & 20 & 80 & 0 & \mathbf{0} \\
0 & 0 & 0 & 80 & \mathbf{0} \\
0 & 0 & 0 & 0 & \mathbf{140} \\
 & & & & \\
\mathbf{60} & \mathbf{40} & \mathbf{0} & \mathbf{40} &
\end{array}\right] \quad , \quad \mathbf{C} = \left[\begin{array}{cccc|c}
\underline{5} & \underline{2} & \underline{3} & \underline{6} & \mathbf{na} \\
\underline{2} & \underline{7} & \underline{7} & \underline{4} & \mathbf{na} \\
1 & 3 & \underline{6} & 9 & \mathbf{2} \\
 & & & & \\
\mathbf{na} & \mathbf{na} & \mathbf{na} & \mathbf{na} &
\end{array}\right] \quad ,
$$

Now we only have one row left, namely the third row, we assign as much as we can, 60, to the cheapest cell, $[3,1]$, as much as we can , 40, (of the remaining 80) to the second cheapest cell, $[3,2]$, and the remaining 40 to cell $[3,4]$. We now have

$$
\left[\begin{array}{cccc|c}
0 & 20 & 80 & 0 & \mathbf{0} \\
0 & 0 & 0 & 80 & \mathbf{0} \\
60 & 40 & 0 & 40 & \mathbf{0} \\
 & & & & \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} &
\end{array}\right] \quad ,
$$

Where all the remaining supply and demand are 0, as they should. Extracting the tableau itself, gives us that the initial basic feasible solution according to Vogel's method is

$$
\begin{bmatrix}
0 & 20 & 80 & 0 \\
0 & 0 & 0 & 80 \\
60 & 40 & 0 & 40
\end{bmatrix}
$$

The **initial cost** is

$$20 \cdot 2 + 80 \cdot 3 + 80 \cdot 4 + 60 \cdot 1 + 40 \cdot 3 + 40 \cdot 9 = 1140 \quad .$$

Note that it is much better than the initial cost of the initial Transportation tableau given by the Minimal Cost Rule, done in the first part of this problem, that happened to be 1580.