Solution to Attendance Quiz for Lecture 14

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1. Suppose that $x_1 = 0$, $x_2 = 2$, $x_3 = 0$ is an optimal solution to the linear programming problem

Maximize $x_1 + 3x_2 + x_3$

subject to

$$x_1 + x_2 + 2x_3 \leq 3$$

$$x_1 + 2x_2 + x_3 \leq 4$$

$$2x_1 + x_2 + x_3 \leq 5$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad .$$

Using the principle of complementary slackness and the duality theorem, find an optimal solution to the dual problem. What value will the objective function of the dual problem have at this optimal solution?

Sol. to 1:

The **dual problem** is

Minimize $3w_1 + 4w_2 + 5w_3$

subject to

$$w_1 + w_2 + 2w_3 \geq 1$$

$$w_1 + 2w_2 + w_3 \geq 3$$

$$2w_1 + w_2 + w_3 \geq 1$$

$$w_1 \geq 0 \quad , \quad w_2 \geq 0 \quad , \quad w_3 \geq 0 \quad .$$

Note that

• The **slack** of the first constraint at the given optimal solution, $x_1 = 0$, $x_2 = 2$, $x_3 = 0$ is $3 - 0 - 2 - 2 \cdot 0 = 1$, this is positive. We know right away that $w_1 = 0$.

• The **slack** of the second constraint at the given optimal solution $x_1 = 0$, $x_2 = 2$, $x_3 = 0$ is $4 - 0 - 2 \cdot 2 - 0 = 0$, this is zero, hence we can't conclude anything about w_2.

- The **slack** of the third constraint at the given optimal solution $x_1 = 0$, $x_2 = 2$, $x_3 = 0$ is $5 - 2 \cdot 0 - 2 - 0 = 3$, this is positive. We know right away that $w_3 = 0$.

Since x_2 is **positive**, we also know that the slack of the second constraint of the dual problem is zero. In other words, it is an **equality**.

$$w_1 + 2w_2 + w_3 = 3$$

Since $w_1 = 0$ and $w_3 = 0$ we get

$$0 + 2w_2 + 0 = 3 \quad ,$$

and conclude that an optimal solution to the dual problem is

$w_1 = 0$, $w_2 = \frac{3}{2}$, $w_3 = 0$.

The **optimal value** is $3 \cdot 0 + 4 \cdot \frac{3}{2} + 5 \cdot 0 = 6$, as it should! Since the optimal value of the primal problem is $0 + 3 \cdot 2 + 0 = 6$.