¿

Solutions to Attendance Quiz for Lecture 12

1. Find the dual of the given linear programming problem.

Minimize $9x_1 + 4x_2 + 3x_3 + 9x_4$
subject to

$$x_1 + 3x_2 + x_3 + x_4 \geq 5 \quad , \quad 4x_1 + x_2 + 6x_3 + 9x_4 \geq 8 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad , \quad x_4 \geq 0 \quad .$$

First Sol. to 1 (the long way): Let's first convert it to **standard form**.

Maximize $z = -9x_1 - 4x_2 - 3x_3 - 9x_4$
subject to

$$-x_1 - 3x_2 - x_3 - x_4 \leq -5 \quad , \quad -4x_1 - x_2 - 6x_3 - 9x_4 \leq -8 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad , \quad x_4 \geq 0 \quad .$$

In **matrix form** the problem is

Maximize $z = \mathbf{c}^T\mathbf{x}$ subject to

$$A\mathbf{x} \leq \mathbf{b} \quad , \quad \mathbf{x} \geq \mathbf{0}$$

where

$$\mathbf{c} = \begin{bmatrix} -9 \\ -4 \\ -3 \\ -9 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} -5 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -3 & -1 & -1 \\ -4 & -1 & -6 & -9 \end{bmatrix}$$

The **dual problem**, still in matrix notation is

Minimize $z' = \mathbf{b}^T\mathbf{w}$ subject to

$$A^T\mathbf{w} \geq \mathbf{c} \quad , \quad \mathbf{w} \geq \mathbf{0}$$

where

$$\mathbf{c} = \begin{bmatrix} -9 \\ -4 \\ -3 \\ -9 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} -5 \\ -8 \end{bmatrix}$$

where

$$A^T = \begin{bmatrix} -1 & -4 \\ -3 & -1 \\ -1 & -6 \\ -1 & -9 \end{bmatrix} \quad .$$

This spells out to

Minimize $z' = -5w_1 - 8w_2$ subject to

$$-w_1 - 4w_2 \geq -9 \quad ,$$

$$-3w_1 - w_2 \geq -4 \quad ,$$

$$-w_1 - 6w_2 \geq -3 \quad ,$$

$$-w_1 - 9w_2 \geq -9 \quad ,$$

$$w_1 \geq 0 \quad , \quad w_2 \geq 0 \quad .$$

Since all the minus signs are annoying, this can be rephrased as follows.

Maximize $z' = 5w_1 + 8w_2$ subject to

$$w_1 + 4w_2 \leq 9 \quad ,$$

$$3w_1 + w_2 \leq 4 \quad ,$$

$$w_1 + 6w_2 \leq 3 \quad ,$$

$$w_1 + 9w_2 \leq 9 \quad ,$$

$$w_1 \geq 0 \quad , \quad w_2 \geq 0 \quad .$$

Second Sol. to 1 (much faster!). Since the "dual of the dual" is the original, we can just use the "dictionary" $max \leftrightarrow min$, $\geq \leftrightarrow \leq$, 'coefficients of goal functions' $\leftrightarrow$ 'coefficients of constraints' to get **immediately**

Maximize $z' = 5w_1 + 8w_2$ (since $5, 8$ are on the right side of the constraints) subject to

$$w_1 + 4w_2 \leq 9 \quad ,$$

(since 9 is the coefficient of x_1 in the original goal function, and $1, 4$ are the coefficients of x_1 in the first and second constrains respectively)

$$3w_1 + w_2 \leq 4 \quad ,$$

(since 4 is the coefficient of x_2 in the original goal function, and $3, 1$ are the coefficients of x_2 in the first and second constrains respectively)

$$w_1 + 6w_2 \leq 3 \quad ,$$

(since 3 is the coefficient of x_3 in the original goal function, and $1, 6$ are the coefficients of x_3 in the first and second constrains respectively)

$$w_1 + 9w_2 \leq 9 \quad ,$$

(since 9 is the coefficient of x_4 in the original goal function, and $1, 9$ are the coefficients of x_4 in the first and second constrains respectively)

$$w_1 \geq 0 \quad , \quad w_2 \geq 0 \quad .$$