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FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
No Calculators! No books! No Notes! To ensure maximum credit, organize your work neatly and be sure to show all your work.
Do not write below this line

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1. 10 (out of 10)

2. 10 (out of 10)

3. 10 (out of 20)

4. 20 (out of 20)

5. 20 (out of 20)

6. 10 (out of 10)

7. 10 (out of 10)

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tot.: (out of 100)

100

EXCELLENT!

1. (10 points) Find a basis for the subspace V, of R^5 (EXPLAIN!)

$$V = Span\{ \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}3\\3\\3\\2\\1\end{bmatrix}, \begin{bmatrix}2\\2\\2\\2\\1\end{bmatrix} \}.$$

Ans. $\boxed{\left\{ \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix} \right\}}$

$$\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} + \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix} + \begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix} = \begin{bmatrix}3\\3\\3\\2\\1\end{bmatrix}$$

So remove linear combination

$$\left\{ \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\2\\2\\2\\1\end{bmatrix} \right\}$$

$$\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix} + \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix} = \begin{bmatrix}2\\2\\2\\2\\1\end{bmatrix}$$

So remove linear combination

$$\left\{ \begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\\0\end{bmatrix} \right\}$$

Since none add up to each other or are multiples of each other, this is final answer.

2. (10 points.) Use any method you wish (but EXPLAIN) to solve the following linear optimization problem. Give the **optimal solution** as well as the **optimal value.**

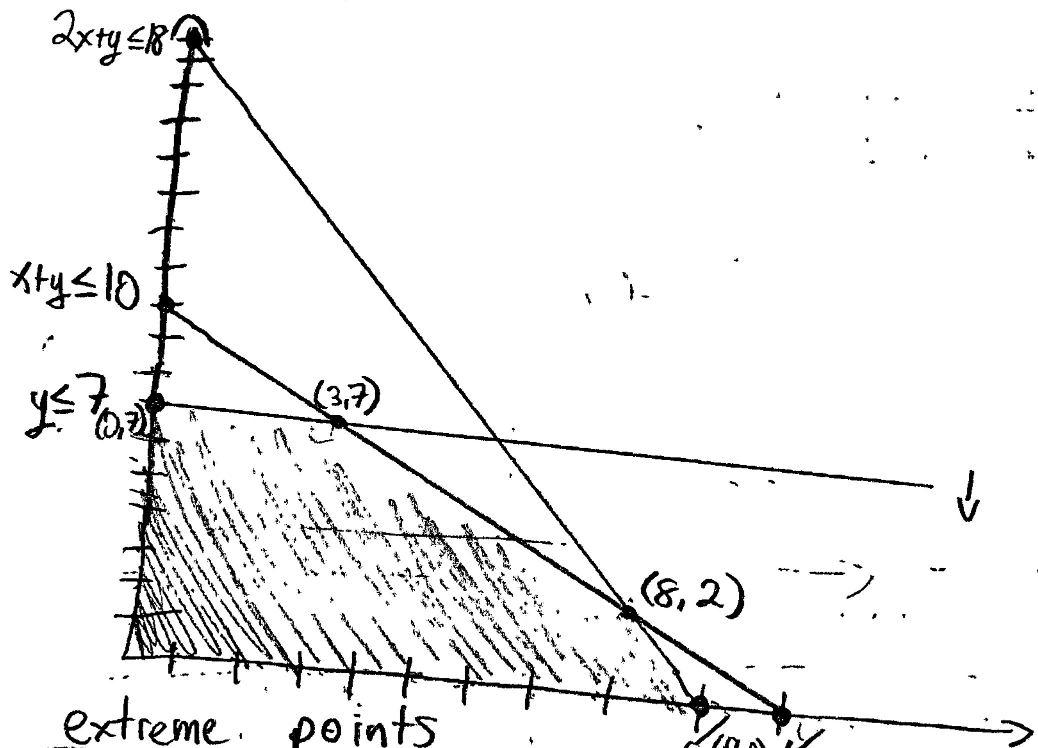
Maximize $z = x + 2y$ subject to the constraints

$$y \leq 7 \quad , \quad x + y \leq 10 \quad , \quad 2x + y \leq 18 \quad ,$$

$$x \geq 0 \quad , \quad y \geq 0 \quad .$$

Ans. The optimal solution is $x =$ 3 and $y =$ 7 . The optimal value is $z =$ 17.

Graph method



extreme points

x	y	z=x+2y
0	7	0+2(7)=14
9	0	9+2(0)=9 max
3	7	3+2(7)=17
8	2	8+2(2)=12

find intersections

$x + 7 = 10$

$x = 3 \Rightarrow (3,7)$

$2x + y = 18$

$-\ 2x + 2y = 20$

$-y = -2$

$y = 2 \rightarrow (8,2)$

$2x + 2 = 18$

$2x = 16$

$x = 8$

(8,2)

3. (20 points.) Consider the following linear programming problem in canonical form. Maximize $z = 2x_1 - x_2 + x_3$ subject to the constraint

$$2x_1 + x_2 + x_3 = 13 \quad , \quad 3x_1 + 2x_2 + x_3 = 18 \quad , \quad x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \ .$$

(i) Find the set of **basic solutions.** For each of them indicate the set of basic variables and the set of non-basic variables.

(ii) Find the set of **basic feasible solutions.**

(iii) Find the optimal solution(s)

(iv) Find the optimal value.

Ans. to (i): The set of **basic solutions** (for each, in parentheses, indicate the set of basic variable) is:

basic basic nonbasic basic basic basic basic

$\{(8,-3,0),(5,0,3),(0,5,8)\}$

Ans. to (ii): The set of **basic feasible solutions** is

basic basic basic basic

$\{(5,0,3);(0,5,8)\}$

Ans to (iii): The **optimal solution** is $x_1 = 5$, $x_2 = 0$, $x_3 = 3$.

Ans. to (iv): The **optimal value** is $z = 13$.

Since there are 2 constraints, we have 2 basic vars

Basic vars	x_1	x_2	x_3	feasible?	$z=2x_1-x_2+x_3$
(x_1,x_2)	8	-3	0	no	N/A since not feasible
(x_1,x_3)	5	0	3	yes	$2(5)-0+3=10+3=13$ max
(x_2,x_3)	0	5	8	yes	$2(0)-5+8=3$

(x_1,x_2)

$2x_1+x_2=13$

$3x_1+2x_2=18$

↓

$4x_1+2x_2=26$

$-3x_1+2x_2=18$

$x_1=8$

$2(8)+x_2=13$

$16+x_2=13$

$x_2=-3$

(x_1,x_3)

$2x_1+x_3=13$

$-\ 3x_1+x_3=18$

$-x_1=-5$

$x_1=5$

⇓

$2(5)+x_3=13$

$10+x_3=13$

$x_3=3$

(x_2,x_3)

$x_2+x_3=13$

$-\ 2x_2+x_3=18$

$-x_2=-5$

$x_2=5$

⇓

$5+x_3=13$

$x_3=8$

4. (20 points.) Use the big M-method (no credit for any other method!) to solve the following Linear Programming problem (that takes a second to do using the graphical method).

Maximize $2x_1 + 3x_2$ subject to the constraints

$$x_1 + x_2 = 5 \quad , \quad x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad .$$

(i) (3 points): Set up an equivalent problem with artificial variable(s). Give the artificial variable(s) name(s). Express the objective function in terms of the natural variables x_1, x_2.

(ii) (7 points) Set up the **initial tableau**, indicating for each row (except for the bottom, objective row) its basic variable.

(iii) (10 points) Use the big M-method to find the optimal solution and the optimal value.

i. Maximize $z = 2x_1 + 3x_2 - My_1$

$x_1 + x_2 + y_1 = 5$

$\Downarrow$

$y_1 = 5 - x_1 - x_2$

substitute new y_1

$Z = 2x_1 + 3x_2 - M(5 - x_1 - x_2)$

$0 = z - 2x_1 - 3x_2 + M(5 - x_1 - x_2) \Rightarrow 0 = z - 2x_1 - 3x_2 + 5M - Mx_1 - Mx_2$

$\Rightarrow -5M = z + x_1(-2-M) + x_2(-3-M)$

ii

basic	x_1	x_2	y_1	z		θ
y_1	1	1	1	0	5	5 ←
	$-2-M$	$-3-M$	0	1	$-5M$	
		↑ most neg.				

iii

	basic	x_1	x_2	y_1	z	
$R_2 \to R_2 + (3+M)R_1$	x_2	1	1	1	0	5
$(-2-M)+(3+M)$		1	0	$3+M$	1	15

$(-5M) + 5(3+M)$

$-5M + 15 + 5M$

15

optimal solution

$$\boxed{x_1 = 0,\ x_2 = 5,\ z = 15}$$

5. (20 points.) Solve the following linear programming problem using the simplex method (no credit for any other method).

Maximize $z = 2x_1 + 5x_2$ subject to

$$3x_1 + 5x_2 \le 8 \quad ,$$
$$2x_1 + 7x_2 \le 12 \quad ,$$
$$x_1 \ge 0 \quad , \quad x_2 \ge 0 \quad .$$

Max $z = 2x_1 + 5x_2$ ⇓ introduce slack variables (convert to canonical)

$3x_1 + 5x_2 + y_1 = 8$

$2x_1 + 7x_2 + y_2 = 12$

$x_1, x_2, y_1, y_2 \ge 0$

convert goal fuction

$0 = z - 2x_1 - 5x_2$ ⇓ put into tableau

basic	x_1	x_2	y_1	y_2	z		θ
y_1	3	5	1	0	0	8 ←	$8/5 - \frac{56}{35}$
y_2	2	7	0	1	0	12 ←	$12/7 - \frac{60}{35}$
	-2	-5	0	0	1	0	

most neg

$r_1 \to \frac{1}{5} r_1$

$r_2 \to r_2 - 7r_1$

$r_3 \to r_3 + 5r_1$

basic	x_1	x_2	y_1	y_2	z	
x_2	$3/5$	1	$1/5$	0	0	$8/5$
y_2	$-11/5$	0	$-7/5$	1	0	$4/5$
	1	0	1	0	1	8

$2 - \frac{21}{5}$

$\frac{10}{5} - \frac{21}{5}$

$12 - \frac{56}{5}$

$\frac{60}{5} - \frac{56}{5}$

$-2 + \frac{15}{5}$

$-\frac{10}{5} + \frac{15}{5} = 1$

$0 + (\frac{8}{5})5 = 0 + \frac{40}{5} = 8$

optimal solution

$$\boxed{x_1 = 0, \; x_2 = 8/5, \; z = 8}$$

6. (10 points)
Set up a linear programming model of the situation described. Determine whether it is in standard form. If not make it standard.

A restaurant chef is planning a meal consisting of two foods, humus, and eggplant salad.

- Each kg of Humus contains 30 grams of fat and 100 grams of protein .
- Each kg of eggplant salad contains 50 grams of fat and 50 grams of protein.

The chef wants the meal to consist of at least 300 grams of protein and at most 200 grams of fat.

If the profit that he makes is 10 dollars per kg for humus and 5 dollars per kg for eggplant salad, how many kilograms of each food should be served so as to maximize ~~her profit~~?

Let ~~M~~ x_1 = hummus, x_2 = eggplant.

	x_1	x_2	limits
fat	30	50	≤ 200
protein	100	50	≥ 300
price	\$10	\$5	max

Maximize $z = 10x_1 + 5x_2$

$30x_1 + 50x_2 \leq 200$

$100x_1 + 50x_2 \geq 300$

$x_1 \geq 0, x_2 \geq 0$

NOT STANDARD FORM

$$\boxed{\begin{aligned} &\text{Max } z = 10x_1 + 5x_2 \\ &30x_1 + 50x_2 \leq 200 \\ &-100x_1 - 50x_2 \leq -300 \\ &x_1 \geq 0, x_2 \geq 0 \end{aligned}}$$

← standard form

7. (10 points) Find the dual of the given linear programming problem.
Maximize $x_1 + 2x_2 + 3x_3$
subject to

becomes resource column

$w_1 \quad w_1 \quad w_1 \quad$ goal $\quad w_2 \quad w_2 \quad w_2 \quad$ goal

$x_1 + 3x_2 + x_3 \leq 5$; $4x_1 + x_2 + 6x_3 = 8$, $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$.

All signs flipped except non-negative constraints (≥ 0)

$$\boxed{\begin{aligned} &\text{Minimize } z' = 5w_1 + 8w_2 \\ &w_1 + 4w_2 \geq 1 \\ &3w_1 + w_2 \geq 2 \\ &w_1 + 6w_2 \geq 3 \\ &w_1 \geq 0, \ w_2 \text{ unconstrained} \end{aligned}}$$

↑ because 2nd equation is an equality, 2nd var is unconstrained