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SCC: (please circle): Only I/Only II/I and II/None

MATH 354 (3), Dr. Z. , Final Exam , Thurs., Dec. 21, 2023, 8:00am-11:00am; place: TIL-246 .

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question)

Show all your work! No calculators, no cheatsheets

EXPLAIN EVERYTHING

CHECK ALL YOUR ANSWERS! (whenever applicable) . If you did it the right way, but got the wrong answer due to a careless computational error, that you could have detected by checking, you will get NO partial credit.

Do not write below this line

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1. (out of 10)
2. (out of 10)
3. -3 (out of 10)
4. (out of 10)
5. (out of 10)
6. (out of 10)
7. (out of 10)
8. (out of 10)
9. (out of 10)
10. (out of 10)
11. (out of 10)
12. (out of 10)
13. (out of 20)
14. (out of 20)
15. (out of 20)
16. -5 (out of 20)

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tot. 192 (out of 200)

1. (10 points) Find the rank of the following matrix. Explain! (No credit for just giving the answer.)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 5 \end{bmatrix}.$$

Ans.: The rank of the matrix is: 5

The number of nonzero rows after row reduction is 5. (And the matrix stays the same due to the fact that all rows are independent of one another).

2. (10 points) Consider the following scenario.

A restaurant chef is planning a meal consisting of three foods: soup, pasta, and pudding.

- The soup has 100 calories per liter, the pasta has 10 calories per kg, and the pudding has 200 calories per kg.

- The soup has 200 grams of protein per liter, the pasta has 50 grams of protein per kg , the pudding has 30 grams of protein per kg.

- The soup has 70 grams of carbohydrates per liter, the pasta has 40 grams of carbohydrates per kg, the pudding has 50 grams of carbohydrates per kg.

The chef wants the meal to consist of

- At least 300 grams of protein.
- At least 400 grams of carbohydrates but at most 600 grams of carbohydrates.
- At most 1000 calories.

The profit that he makes is 2 dollars per liter of soup, 3 dollars per kg of pasta, and 5 dollars per kg of pudding.

Set up, but do **not** solve, a linear programming model of the situation described that would maximize the profits subject to the constraints. Call z the profit and x_1 the amount of soup in liters, x_2 the amount of pasta in kg, and x_3 the amount of pudding in kg.

Ans.:

Maximize $z = 2x_1 + 3x_2 + 5x_3$ sub. to

$$100x_1 + 10x_2 + 200x_3 \leq 1000$$
$$200x_1 + 50x_2 + 30x_3 \geq 300$$
$$70x_1 + 40x_2 + 50x_3 \geq 400$$
$$70x_1 + 40x_2 + 50x_3 \leq 600$$
$$x_1, x_2, x_3 \geq 0$$

	Soup	Pasta	Pudding		
Calories	100	10	200	≤ 1000	
Protein	200	50	30	≥ 300	
Carbs	70	40	50	≥ 400	≤ 600
Profit	2	3	5		

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3 (10 points) Convert the following linear programming problem into canonical form, by introducing more variables, if necessary. Explain what you are doing. Just giving the right answer, without explanation will earn you only two points).

Minimize $z = 4x_1 + 3x_2 - 4x_3$

Subject to

$$x_1 - x_2 + x_3 \geq 5$$

$$2x_1 - 3x_2 - x_3 \leq 5$$

$x_1 \geq 0$, $x_2 \leq 0$, x_3 unrestricted.

10

Ans.:

maximize $z = -4x_1 + 3x_2' + 4x_3' - 4x_3''$

subject to $-x_1 - x_2' - x_3' + x_3'' + x_4 = -5$

$2x_1 + 3x_2' - x_3' + x_3'' + x_5 = 5$

$x_1, x_2', x_3', x_3'', x_4, x_5 \geq 0$

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let $x_3 = x_3' - x_3''$ s.t. $x_3', x_3'' \geq 0$.

let $x_2 = -x_2'$ s.t. $x_2' \geq 0$

minimize $z = 4x_1 - 3x_2' - 4x_3' + 4x_3''$

subject to $x_1 + x_2' + x_3' - x_3'' \geq 5$

$2x_1 + 3x_2' - x_3' + x_3'' \leq 5$

$x_1, x_2', x_3', x_3'' \geq 0$

standard form:

maximize $z = -4x_1 + 3x_2' + 4x_3' - 4x_3''$ ← this new z is the negative of the old z

subject to $-x_1 - x_2' - x_3' + x_3'' \leq -5$

$2x_1 + 3x_2' - x_3' + x_3'' \leq 5$

$x_1, x_2', x_3', x_3'' \geq 0$

canonical form (introduce slack variables):

maximize $z = -4x_1 + 3x_2' + 4x_3' - 4x_3'$

subject to $-x_1 - x_2' - x_3' + x_3'' + x_4 = -5$

$2x_1 + 3x_2' - x_3' + x_3'' + x_5 = 5$

$x_1, x_2', x_3', x_3'', x_4, x_5 \geq 0$

4. (10 points) Use Vogel's method (no credit for other ways!) to find an initial feasible solution to the following transportation problem. Also find its cost. Explain!

$$\mathbf{C} = \begin{bmatrix} 4 & 6 & 7 & 5 \\ 4 & 4 & 7 & 8 \\ 5 & 3 & 6 & 5 \\ 6 & 5 & 3 & 4 \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} 100 \\ 60 \\ 50 \\ 70 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} 40 \\ 70 \\ 120 \\ 50 \end{bmatrix}.$$

280   280

Ans.

$$\begin{bmatrix} 40 & 0 & 10 & 50 \\ 0 & 20 & 40 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 70 & 0 \end{bmatrix}; \quad Cost = \$\,1200$$

Difference Table:

4	6	7	5	1 1 1
4	4	7	8	0 3
5	3	6	5	2
6	5	3	4	1
0	1	3	1	
0	1	3		
	0			

40	0	10	50	100
0	20	40	0	60
0	50	0	0	50
0	0	70	0	70
40	70	120	50	

$$Cost = 40 \cdot 4 + 10 \cdot 7 + 50 \cdot 5 + 20 \cdot 4 + 40 \cdot 7 + 50 \cdot 3 + 70 \cdot 3$$

$$= 160 + 70 + 250 + 80 + 280 + 150 + 210$$

$$= 230 + 330 + 430 + 210$$

$$= 560 + 640$$

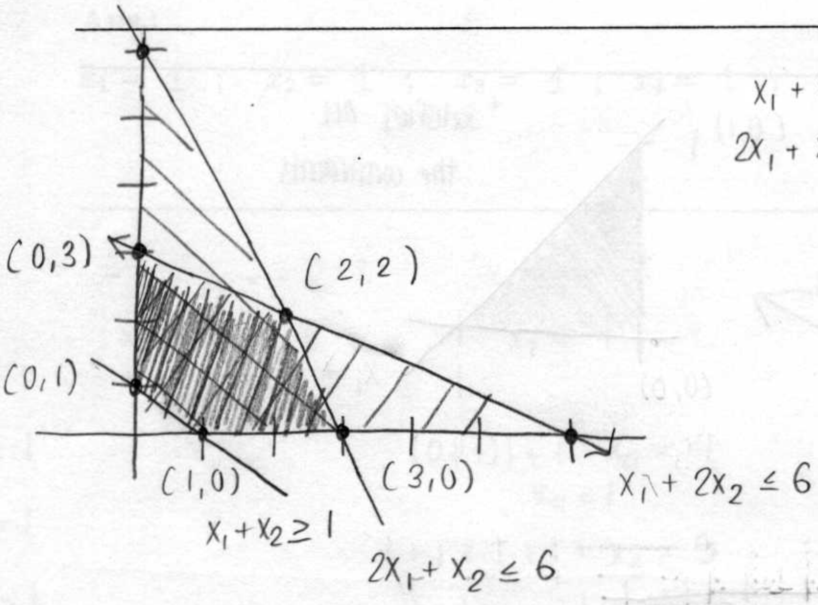
$$= 1200$$

5. (10 points) Use any method to solve the following linear programming problem.

Maximize $z = 3x_1 + 5x_2$, subject to the constraints

$$x_1 + 2x_2 \le 6 \ ,$$
$$2x_1 + x_2 \le 6 \ ,$$
$$x_1 + x_2 \ge 1 \ ,$$
$$x_1 \ge 0 \ , \quad x_2 \ge 0 \ .$$

Ans.: $x_1 =$ 2 ; $x_2 =$ 2 ; $z =$ 16 .



$X_1 + 2X_2 = 6$
$2X_1 + X_2 = 6$

$-2X_1 - 4X_2 = -12$
$2X_1 + X_2 = 6$
$-3X_2 = -6$
$X_2 = 2$

$X_1 + 2(2) = 6$
$X_1 = 2$

Final Candidates

X_1	X_2	Max $z = 3X_1 + 5X_2$
0	3	15
0	1	5
1	0	3
3	0	9
2	2	6 + 10 = 16

+

6. (10 points) Use any method to solve the following linear programming problem.

Minimize $z = 3\,x_1 - 5\,x_2$, subject to the constraints

Max $z = -3x_1 + 5x_2$

$$5\,x_1 + x_2 \le 50 \ ,$$

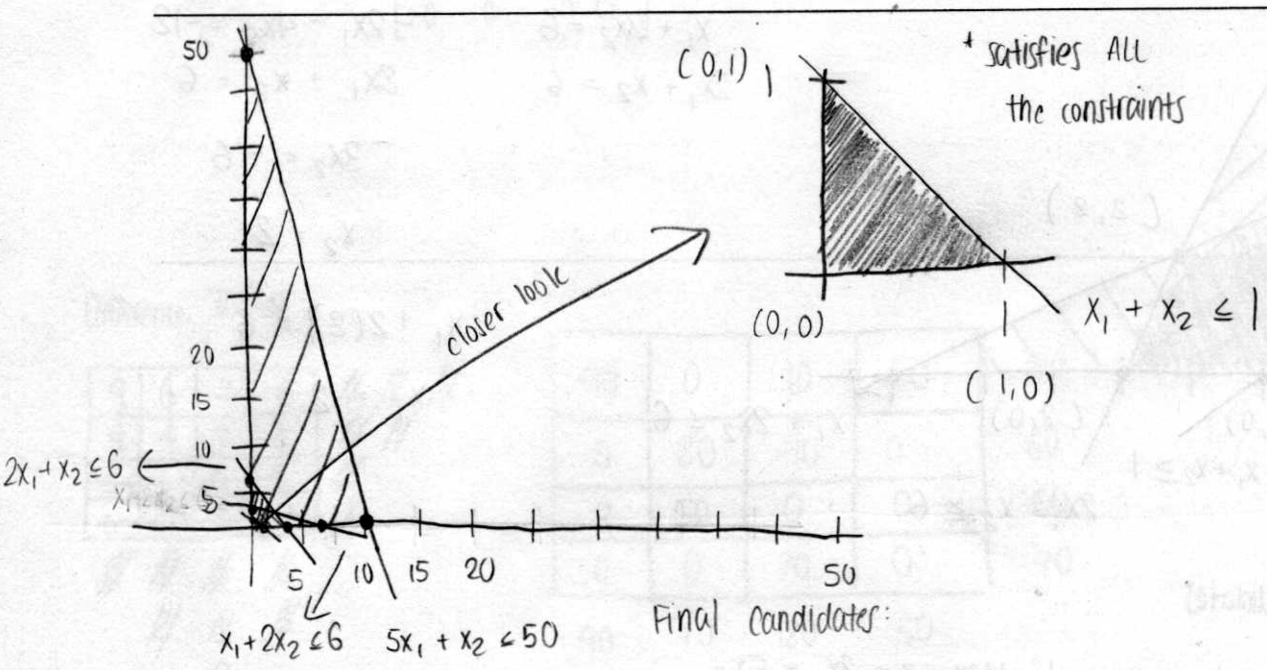
$$x_1 + 2\,x_2 \le 6 \ ,$$

$$2\,x_1 + x_2 \le 6 \ ,$$

$$x_1 + x_2 \le 1 \ ,$$

$$x_1 \ge 0 \ , \quad x_2 \ge 0 \ .$$

Ans.: $x_1 =$ 0 ; $x_2 =$ 1 ; $z =$ -5 .



Final candidates:

x_1	x_2	Min $z = 3x_1 - 5x_2$
0	0	0
0	1	-5
1	0	3

7. (10 points) Solve the following linear programming problem

Minimize $z = 3 + x_1 + x_2 + x_3 - x_4 - x_5$ subject to the constraints:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 5 \quad ,$$

$$x_1 + x_2 + x_3 + x_4 = 4 \quad ,$$

$$x_1 + x_2 + x_3 = 3 \quad ,$$

$$x_1 + x_2 = 2 \quad ,$$

$$x_1 + 2x_2 = 3 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad , \quad x_4 \geq 0 \quad , \quad x_5 \geq 0 \quad .$$

Ans.:

$x_1 =$ 1 ; $x_2 =$ 1 ; $x_3 =$ 1 ; $x_4 =$ 1 ; $x_5 =$ 1 ; $z =$ 4 .

$-x_1 - x_2 = -2$

$x_1 + 2x_2 = 3$

$x_2 = 1$

$x_1 + 1 = 2$

$x_1 = 1$

$1 + 1 + x_3 = 3$

$x_3 = 1$

$1 + 1 + 1 + x_4 = 4$

$x_4 = 1$

$1 + 1 + 1 + 1 + x_5 = 5$

$x_5 = 1$

$z = 3 + 1 + 1 + 1 - 1 - 1$

$= 4$

$x_1 = 1$

$x_2 = 1$

$x_3 = 1$

$x_4 = 1$

$x_5 = 1$

$z = 4$

8. (10 points altogether) Consider the following linear programming problem in canonical form.

Maximize $z = x_1 + 2\,x_2 + 3\,x_3 + 4\,x_4$ subject to the constraints:

$$x_1 + x_2 + x_3 + x_4 = 2 \quad ,$$

$$2x_1 + x_2 + x_3 + x_4 = 3 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad , \quad x_4 \geq 0 \quad .$$

(i) (4 points) Find the set of **basic solutions.** For each of them indicate the set of basic variables and the set of non-basic variables.

(ii) (2 points) Find the set of **basic feasible solutions.**

(iii) (2 points) Find the optimal solution(s)

(iv) (2 points) Find the optimal value.

Ans. to (i): The set of **basic solutions** (for each, in parentheses, indicate the set of basic variable) is:

(x_1, x_2), (x_1, x_3), (x_1, x_4), (x_2, x_3), (x_2, x_4), (x_3, x_4)

Ans. to (ii): The set of **basic feasible solutions** is

(x_1, x_2), (x_1, x_3), (x_1, x_4)

Ans to (iii): An **optimal solution** is $x_1 =$ 1 , $x_2 =$ 0 , $x_3 =$ 0 , $x_4 =$ 1

.

Ans. to (iv): The **optimal value** is $z =$ 5 .

Basic	x_1	x_2	x_3	x_4	feasible?	Max $z = x_1 + 2x_2 + 3x_3 + 4x_4$
(x_1, x_2)	1	1	0	0	yes	3
(x_1, x_3)	1	0	1	0	yes	4
(x_1, x_4)	1	0	0	1	yes	5
(x_2, x_3)	0	NA	NA	0	no	–
(x_2, x_4)	0	NA	0	NA	no	–
(x_3, x_4)	0	0	NA	NA	no	–

$x_1 + x_2 = 2$
$2x_1 + x_2 = 3$

$-x_1 - x_2 = -2$
$2x_1 + x_2 = 3$
$x_1 = 1 \quad x_2 = 1$

$x_1 + x_3 = 2$
$2x_1 + x_3 = 3$
$x_1 = 1 \quad x_3 = 1$

$x_1 + x_4 = 2$
$2x_1 + x_4 = 3$

$x_2 + x_3 = 2$
$x_2 + x_3 = 3$
$2 \neq 3$

$x_2 + x_4 = 2$
$x_2 + x_4 = 3$
$2 \neq 3$

9. (10 points) Suppose that $x_1 = 2$, $x_2 = 0$, $x_3 = 4$ is an optimal solution to the following linear programming problem.

Maximize $z = 4x_1 + 2x_2 + 3x_3$,

subject to:

$$2x_1 + 3x_2 + x_3 \leq 12 \quad ,$$

$$x_1 + 4x_2 + 2x_3 \leq 10 \quad ,$$

$$3x_1 + x_2 + x_3 \leq 10 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad .$$

Using the principle of complementary slackness (**no credit for other methods!**), and the duality theorem, find an optimal solution to the dual problem (calling the dual variables w_1, w_2, w_3). What value will the objective function of the dual problem have at the optimal solution? **Explain everything!** (only 2 points for the right answers without explanations).

Ans.: $w_1 =$ 0 ; $w_2 =$ 1 ; $w_3 =$ 1 ; optimal value of dual (z') = 20 .

Since x_1 and x_3 are nonzero, this means the first and third constraint of the dual problem has to be an equality.

$2(2) + 3(0) + 4 = 8 \leq 12$, slack means $w_1 = 0$.

$2 + 4(0) + 2(4) = 10 \leq 10$, we cannot conclude anything } no slack

$3(2) + 0 + 4 = 10 \leq 10$, we cannot conclude anything

$z = 4(2) + 2(0) + 3(4) = 8 + 12 = 20$ (This means by the duality thrm, the opt. value of the dual is also 20)

Dual: Min $z' = 12w_1 + 10w_2 + 10w_3$

$2w_1 + w_2 + 3w_3 = 4$

$3w_1 + 4w_2 + w_3 \geq 2$

$w_1 + 2w_2 + w_3 = 3$

$w_1 = 0$

$w_2 + 3w_3 = 4$

$2w_2 + w_3 = 3$

$-2w_2 - 6w_3 = -8$

$2w_2 + w_3 = 3$

$-5w_3 = -5$

$w_3 = 1$

$w_2 + 1(3) = 4$

$w_2 = 1$

10. (10 points) In the course of solving an assignment problem with five employees and five jobs, the following partial solution was arrived at:

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0 \\ 3 & 2 & 0 \leftarrow & 1 & \!\!-\!\!\lrcorner\, 0^* \\ 0 \leftarrow & 1 & \!\!-\,0^* \downarrow & 1 & 1 \\ 0^* \downarrow & \!\!-\,0 \rightarrow & 1 & 3 & 2 \\ 2 & 0 & \!\!-\,1\,- & 0^* & 9 \end{bmatrix}.$$

By findind an **alternating path** (no credit for inspection or other methods!), find the permutation, P, that is the final solution. Explain! (No credit without explanation).

Ans.:

$$P = \begin{bmatrix} 1 & 1 & 2 & 1 & 0^+ \\ 3 & 2 & 0^+ & 1 & 0 \\ 0^+ & 1 & 0 & 1 & 1 \\ 0 & 0^+ & 1 & 3 & 2 \\ 2 & 0 & 1 & 0^+ & 9 \end{bmatrix} \quad 53124$$

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0 \\ 3 & 2 & 0 \longleftarrow & \downarrow 0^+ \\ 0 \longleftarrow & & 0^* \downarrow & 1 & 1 \\ 0^* \downarrow & \!\!-\,0 \rightarrow & 1 & 3 & 2 \\ 2 & 0 & 1 & 0^* & 9 \end{bmatrix}$$

alternate the stars (give stars to those w/ no stars, and is a part of the alternating path) $\longrightarrow$

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0^+ \\ 3 & 2 & 0^+ & 1 & 0 \\ 0^+ & 1 & 0 & 1 & 1 \\ 0 & 0^+ & 1 & 3 & 2 \\ 2 & 0 & 1 & 0^+ & 9 \end{bmatrix}$$

11. (10 points altogether) (a) (8 points) Use the **Minimal Cost Rule** (no credit for other ways!) to find an initial feasible solution to the following transportation problem. (b) (2 points) Find its cost.

$$\mathbf{C} = \begin{bmatrix} 4 & 6 & 7 & 5 \\ 4 & 4 & 7 & 8 \\ 5 & 3 & 6 & 5 \\ 6 & 5 & 3 & 4 \end{bmatrix}, \quad \mathbf{s} = \begin{bmatrix} 100 \\ 60 \\ 50 \\ 70 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} 40 \\ 70 \\ 120 \\ 50 \end{bmatrix}.$$

280 280

Ans.

$$\begin{bmatrix} 40 & 10 & 0 & 50 \\ 0 & 60 & 0 & 0 \\ 0 & 0 & 50 & 0 \\ 0 & 0 & 70 & 0 \end{bmatrix}; \quad Cost = \$\,1220.$$

a)

4: 40	6: 10	7: 0	5: 50	100
4: 0	4: 60	7: 0	8: 0	60
5: 0	3: 0	6: 50	5: 0	50
6: 0	5: 0	3: 70	4: 0	70
40	70	120	50	

b) Cost $= 40\cdot 4 + 10\cdot 6 + 5\cdot 50 + 60\cdot 4 + 6\cdot 50 + 3\cdot 70 =$

$160 + 60 + 250 + 240 + 300 + 210$

$= 220 + 490 + 510$

$= 220 + 1000$

$= 1220$

12. (10 points altogether)
(a) (8 points) Solve the assignment problem with the following cost matrix. (b) (2 points) Find the minimal cost.

$$\begin{bmatrix} 3 & 5 & 4 & 2 & 8 & 1 \\ 8 & 3 & 6 & 6 & 4 & 3 \\ 4 & 4 & 8 & 8 & 3 & 5 \\ 3 & 8 & 7 & 4 & 9 & 7 \\ 7 & 7 & 9 & 2 & 3 & 5 \\ 9 & 7 & 2 & 7 & 5 & 8 \end{bmatrix}$$

Ans.:

$$P = \begin{bmatrix} 2 & 4 & 3 & 1 & 7 & 0^+ \\ 5 & 0^+ & 3 & 3 & 1 & 0 \\ 1 & 1 & 5 & 5 & 0^+ & 2 \\ 0^+ & 5 & 4 & 1 & 6 & 4 \\ 5 & 5 & 7 & 0^+ & 1 & 3 \\ 7 & 5 & 0^+ & 5 & 3 & 6 \end{bmatrix} \quad ; \quad Minimal \quad Cost = \$14 .$$

625143

a)

$$\begin{bmatrix} 2 & 4 & 3 & 1 & 7 & 0^+ \\ 5 & 0^+ & 3 & 3 & 1 & 0 \\ 1 & 1 & 5 & 5 & 0^+ & 2 \\ 0^+ & 5 & 4 & 1 & 6 & 4 \\ 5 & 5 & 7 & 0^+ & 1 & 3 \\ 7 & 5 & 0^+ & 5 & 3 & 6 \end{bmatrix}$$

b) $1+3+3+3+2+2 =$ 14

13. (20 points altogether) The capacity matrix for the following network with 7 vertices, (where vertex 1 is the source and vertex 7 is the sink (aka terminal)).

$$\mathbf{C} = \begin{bmatrix} 0 & 8 & 7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 & 0 & 0 & 4 \\ 0 & 0 & 0 & \not{8} & 0 & 0 & 0 \end{bmatrix}.$$

0

(a) (2 points) Draw a diagram of the network with vertex 1 on the left, followed by vertices 2 and 3 (with vertex 2 located above vertex 3), followed by vertex 4 at the middle, followed by vertices 5 and 6 (with 5 located above 6), followed by vertex 7 on the extreme right.

(b) (10 points) Find the maximal flow on the diagram. You don't have to redraw the diagram at every iteration, but indicate each augmenting path that you are using.

(c) (2 points) Find the value of the maximal flow that you found in part (b).

(d) (5 points) Find a minimum cut

(e) (1 point) Find the value of the minimum cut.

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a) Drawing below $\qquad$ 8+5=13 $\qquad$ b) {1→2, 1→3}

Ans.: (c) Value of maximal flow= 13 ;

(d) Minimal cut: The following set of edges: {5→7, 4→7, 6→7} ;

(e) Value of minimal cut = 2+7+4 = 13

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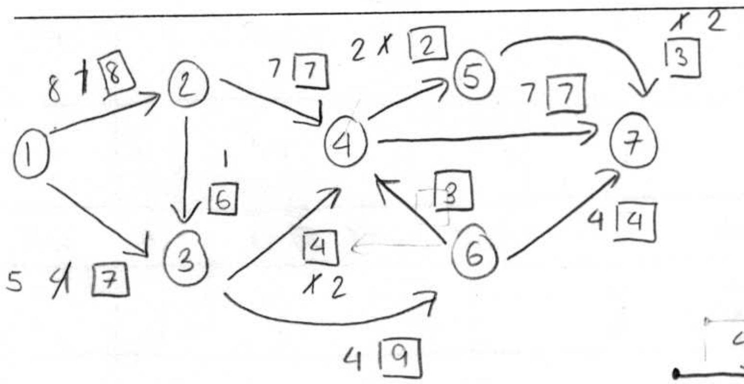
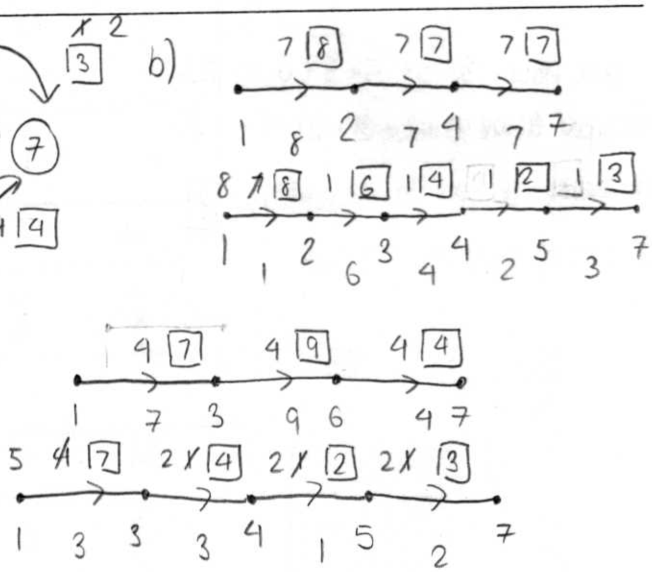
a)



b)



Maximal flow: The set of edges:

{1→2, 1→3}

14. (20 points altogether) In the course of using the "keep finding an augmented path until none are left" algorithm to solve the maximum flow problem for a certain network, the following augmented path was found, from the source, vertex 1, to the sink, vertex 9, where the current flow along an edge is indicated in *italics* and its capacity is indicated in **boldface**

$$1 \overset{\mathit{3},\mathbf{7}}{\longrightarrow} 3 \overset{\mathit{5},\mathbf{11}}{\longrightarrow} 2 \overset{\mathit{5},\mathbf{5}}{\longleftarrow} 7 \overset{\mathit{3},\mathbf{5}}{\longleftarrow} 5 \overset{\mathit{6},\mathbf{17}}{\longrightarrow} 9 \quad .$$

(a) (2 points) Explain why this is a legal augmenting path.
(b) (16 points) Find the new flow along each edge.
(c) (2 points) By how much did it increase?

Ans.:

$$1 \overset{6,\mathbf{7}}{\longrightarrow} 3 \overset{8,\mathbf{11}}{\longrightarrow} 2 \overset{2,\mathbf{5}}{\longleftarrow} 7 \overset{0,\mathbf{5}}{\longleftarrow} 5 \overset{9,\mathbf{17}}{\longrightarrow} 9 \quad .$$

The flow increased by: 3

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a) All the forward-facing arrows' flows are below the capacity, and the backward arrows' flows are nonzero.

b) 1 →(6 ~~3~~ [7]) 3 →(8 ~~5~~ [11]) 2 ←(2 ~~5~~ [5]) 7 ←(0 ~~3~~ [5]) 5 →(9 ~~6~~ [17]) 9

4 6 5 (3) 11

c) Old flow: 3+5+5+3+6 = 22
New flow: 6+8+2+0+9 = 25

25-22 = [3]

15. (20 points altogether) For the following linear programming problem

Maximize $z = 2x_1 + x_2$ subject to the restrictions

$$x_1 + 2x_2 \le 10$$

$$2x_1 + x_2 \le 5$$

$$x_1 \ge 0 \quad , \quad x_2 \ge 0$$

(a) (10 points) Set up an **initial simplex tableau** . Call the slack variables x_3 and x_4.

(b) (2 points) Find the **entering variable** (Explain!).

(c) (2 points) Find the **departing variable** (Explain!).

(d) (6 points) Perform **one** iteration in the simplex algorithm to get the next tableau.

Ans. to (b) and (c) : Entering Variable= X_1 ; Departing Variable= X_4 .

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a) Max $z = 2X_1 + X_2$ sub. to

$X_1 + 2X_2 + X_3 = 10$

$2X_1 + X_2 + X_4 = 5$

$X_1, X_2, X_3, X_4 \ge 0$

Basic	X_1	X_2	X_3	X_4	z	RC
X_3	1	2	1	0	0	10
X_4	2	1	0	1	0	5
	-2	-1	0	0	1	0

$r_2 \to \frac{1}{2} r_2$

b) The entering var. is X_1 since that is the most negative # in the objective function.

$\alpha = \frac{10}{1} = 10$

$\alpha = \frac{5}{2} = 2.5$

c) The departing var is X_4 since the α of X_4 is lower than the α of X_3 ($2.5 < 10$)

d)

Basic	X_1	X_2	X_3	X_4	z	RC
X_3	1	2	1	0	0	10
X_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{5}{2}$
	-2	-1	0	0	1	0

$r_1 \to -r_2 + r_1$

$r_3 \to 2r_2 + r_3$

$\frac{20}{2} - \frac{5}{2} = \frac{15}{2}$

Basic	X_1	X_2	X_3	X_4	z	RC
X_3	0	$\frac{3}{2}$	1	$-\frac{1}{2}$	0	$\frac{15}{2}$
X_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{5}{2}$
	0	0	0	1	1	5

BY ALEXANDRA MARIS

16. (20 points altogether) Consider the following linear programming problem in canonical form, with variables x_1, x_2, x_3, x_4.

Maximize $z = x_1 + 3x_2$ subject to

$$x_1 + x_2 + x_3 = 4 \quad ,$$

$$x_1 + x_2 \quad - x_4 = 6 \quad ,$$

$$x_1 \geq 0 \quad , \quad x_2 \geq 0 \quad , \quad x_3 \geq 0 \quad , \quad x_4 \geq 0 \quad .$$

(a) (4 points) Explain why the usual simplex method would not work.

(b) (2 point) How many artificial variables are needed for the big M method?

(c) (6 points) Calling the artificial variable(s) y_1 (or y_1, y_2 etc., if needed), set up the **initial tableau** for using the big M method.

(d) (4 points) Determine the entering and departing variable of the initial tableau.

(e) (3 points) Perform **one** iteration to get the next tableau.

(f) (1 point) Who is the entering variable right now?

(a) For the usual simplex to work, we need each constraint to have a basic variable (when the problem is in canonical form). For each constraint, the basic variable needs to appear only in that constraint, and have a positive coefficient. Since the RHS needs to be positive, we cannot multiply by −1. In the 2nd constraint, there is no basic variable since the coefficient on x_4 is negative (and x_1, x_2 do not qualify since they aren't exclusive to the 2nd constraint). So we need an artificial variable (for the 2nd constraint). ✓

(b) we need one artificial variable (for the 2nd constraint). ✓

(c) maximize $z = x_1 + 3x_2 - My_1$

subject to $x_1 + x_2 + x_3 = 4$ $\quad x_1, x_2, x_3, x_4, y_1 \geq 0$

$x_1 + x_2 - x_4 + y_1 = 6$

$-x_1 - 3x_2 + M(6 - x_1 - x_2 + x_4) + z = 0$

$-x_1 - 3x_2 + 6M - Mx_1 - Mx_2 + Mx_4 + z = 0$

$(-1-M)x_1 + (-3-M)x_2 + Mx_4 + z = -6M$ ✓

[see page 2 for rest of work]

(20)

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#16

basic	x_1	x_2	x_3	x_4	y_1	z	RHS
← x_3	1	[1]	1	0	0	0	4
y_1	1	1	0	-1	1	0	6
	-1-M	-3-M	0	M	0	1	-6M

θ ratios:
4/1 = 4
6/1 = 6

(d) entering variable: x_2 (most negative objective row entry in column)
departing variable: x_3 (smallest θ ratio since 4 < 6)

(e)

basic	x_1	x_2	x_3	x_4	y_1	z	RHS
x_2	1	1	1	0	0	0	4
y_1	0	0	-1	-1	1	0	2
	2	0	3+M	M	0	1	12-2M

$_2 \to r_2 - r_1$

$_3 \to r_3 + (3+M)r_1$

(f) There is no entering variable because the algorithm is complete (the objective row entries corresponding to variables are all non-negative)