Solutions to Dr. Z.'s Math 354 REAL Quiz #4 (Using the algebraic approach of section 1.5)

1. (8 pts.) (a) Find the extreme points of the set of feasible solutions for the following linear programming problem (b) Find the optimal solution(s)

Minimize $z = 5x - 3y$ subject to

$$x + 2y \leq 4 \quad , \quad x + y \geq 3 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad .$$

Sol. of 1. We first convert to **standard form**

Maximize $z = -5x + 3y$ subject to

$$x + 2y \leq 4 \quad , \quad -x - y \leq -3 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad .$$

Next, we set-up the canonical form

Maximize $z = -5x + 3y + 0 \cdot u + 0 \cdot v$ subject to

$$x + 2y + u = 4 \quad , \quad -x - y + v = -3 \quad , \quad x \geq 0 \quad , \quad y \geq 0 \quad , \quad u \geq 0 \quad , \quad v \geq 0 \quad .$$

• Non-Basic variables $\{x, y\}$, hence **basic variables** $\{u, v\}$. Setting $x = 0, y = 0$ we get the system $u = 4, v = -3$. Since v is negative, this point is **not feasible**.

• Non-Basic variables $\{x, u\}$, hence **basic variables** $\{y, v\}$. Setting $x = 0, u = 0$ we get the

$$2y = 4 \quad , \quad -y + v = -3$$

giving $y = 2$ and $v = -1$. Since v is negative, this point is **not feasible**.

• Non-Basic variables $\{x, v\}$, hence **basic variables** $\{y, u\}$. Setting $x = 0, v = 0$ we get the

$$2y + u = 4 \quad , \quad -y = -3 \quad ,$$

giving $y = 3$ and $u = -2$. Since u is negative, this point is **not feasible**.

• Non-Basic variables $\{y, u\}$, hence **basic variables** $\{x, v\}$. Setting $y = 0, u = 0$ we get the

$$x = 4 \quad , \quad -x + v = -3$$

giving $x = 4$ and $v = 1$. Since these are both non-negative, this point is a feasible solution. Hence $(x, y, u, v) = (4, 0, 0, 1)$ is a feasible extrereme point of the canonical form, and its truncation $(x, y) = (4, 0)$ is an extreme point of the original problem.

• Non-Basic variables $\{y, v\}$, hence **basic variables** $\{x, u\}$. Setting $y = 0, v = 0$ we get the

$$x + u = 4 \quad , \quad -x = -3$$

giving $x = 3$ and $u = 1$. Since these are both non-negative, this point is a feasible solution. Hence $(x, y, u, v) = (3, 0, 1, 0)$ is a feasible extrereme point of the canonical form, and its truncation $(x, y) = (3, 0)$ is an extreme point of the original problem.

• Non-Basic variables $\{u, v\}$, hence **basic variables** $\{x, y\}$. Setting $u = 0, v = 0$ we get the

$$x + 2y = 4 \quad , \quad -x - y = -3 \quad ,$$

giving $x = 2$ and $y = 1$. Since these are both non-negative, this point is a feasible solution. Hence $(x, y, u, v) = (2, 1, 0, 0)$ is a feasible extrereme point of the canonical form, and its truncation $(x, y) = (2, 1)$ is an extreme point of the original problem.

The set of feasible points (of the original problem) is hence $\{(3, 0), (4, 0), (2, 1)\}$.

We plug-in values into $z(x, y) = -5x + 3y$ (recall that we are doing the standard form not the original problem)

•:For $(x, y) = (3, 0)$, we have $z(3, 0) = -5 \cdot 3 + 3 \cdot 0 = -15 \quad ,$

•:For $(x, y) = (4, 0)$, we have $z(4, 0) = -5 \cdot 4 + 3 \cdot 0 = -20 \quad ,$

•:For $(x, y) = (2, 1)$, we have $z(2, 1) = -5 \cdot 2 + 3 \cdot 1 = -7 \quad .$

The largest value is at $x = 2, y = 1$ giving the value -7.

Answer to 1.: The solution is $x = 2, y = 1$ with optimal value (for the standard form) is $z = -7$.

[Note that the optimal value for the **original** problem is $z = 7$, of course the optimal solution does not change, only the value, since when we convert from a "minimize" problem to a "maximize" problem the goal function changes sign].