Solutions to REAL Quiz #7 (Dr. Z., Math 250)

1. (3 points) Find a basis for the following subspace

$$\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3 \, : \, x_1 \, - \, x_2 \, - 2\, x_3 \, = \, 0 \quad \right\}$$

Sol. of 1: We express x_1 in terms of x_2, x_3, and the latter (being free) in terms of themselves, getting the **general solution**

$$x_1 = x_2 + 2x_3 \quad ,$$

$$x_2 = x_2 \quad ,$$

$$x_3 = x_3 \quad ,$$

In **vector notation** this is:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_2 + 2x_3 \\ x_2 \\ x_3 \end{bmatrix} \quad .$$

Separating out the free variables x_2, x_3, we get that this equals

$$x_2 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \quad .$$

The vectors that show up above constitute the desired basis:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad , \quad \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right\} \quad .$$

This is the **answer** .

2. (5 points, 1 each) Deteremine whether the following statements are True or False. Give a short expalanation (or an example or counter-example) in each case. (**No credit without explanation**).

(a) Every non-zero subspace of R^n has a unique basis.

Sol. to 2a): **False**. It has **infintely** many bases.

(b) If $\mathcal{S}$ is a finite subset of V and Span $\mathcal{S}$=V, then $\mathcal{S}$ is a basis for V.

Sol. to 2b): **False**. Span $\mathcal{S}=V$ means that $\mathcal{S}$ is a **generating set** for V, but we are not told that $\mathcal{S}$ is linearly independent, so we can't tell whether or not it is a basis. The corrected statement is

'If $\mathcal{S}$ is a **lineary independnet** subset of V and Span $\mathcal{S}$=V, then $\mathcal{S}$ is a basis for V.'

(c) The dimension of R^{12} is 13.

Sol. to 2c): **False**. The dimension (in the abstract sense) of R^n is always n since the set of **standard vectors** $\{\mathbf{e_1}, \ldots, \mathbf{e_n}\}$ is a basis, and it has n members. So the corrected statement is 'The dimension of R^{12} is 12.'

(d) The columns of any matrix form a basis for its column space.

Sol. to 2d): **False**. they form a **generating set** of the column space. Another way to correct this statement is that the set of **pivot** columns (of the original matrix, i.e. the one **corresponding** to the columns of the reduced-row-echelon form, form a basis for its column space.

(e) The pivot columns of the reduced row echelon form of A always form a basis for its column space.

Sol. to 2e): **False**. The **corresponding** pivot columns of the **original matrix** A form a basis, but **not** the actual columns of the reduced row echelon form that are always standard vectors, and give rise to a very limited repertoire of subspaces.