Solutons to the REAL Quiz #4 (Dr. Z., Math 250)

1. (3 pts.) Find an elemenary matrix E such that $EA = B$, where A and B are as follows:

$$A = \begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ -1 & 1 & 6 & 1 \end{bmatrix} \quad and \quad B = \begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & -1 & 0 & 1 \\ 0 & 3 & 4 & 2 \end{bmatrix}$$

Sol. of 1: We have to decide which **elementary row operation**, when applied to A yields B. The first two rows are exatly the same, so this means that r_3 has been changed. Obviously the new r_3 is not a multiple of the old one, so this means that the new r_3 is the old r_3 plus (or minus) a multiple of either r_1 or r_2. Since the difference between the new r_3 and the old r_3 is $[1, 2, -2]$, that is exactly r_1, the elementary row operation that we need is $r_3 + r_1 \to r_3$.

Now we apply this **very same** elementary row operation to the 3×3 identity matrix, getting

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \overset{r_3 + r_1 \to r_3}{\to} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad .$$

This is the desired matrix E. You are welcome to check, by doing the matrix multiplication, that indeed $EA = B$.

Ans.:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad .$$

2. (3 pts.) Determine whether the following matrix is invertible, and if it is, finds its inverse

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$$

Sol. of 2: We first bring it to **row-echelon form**.

$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \overset{r_2 - \frac{3}{2}r_1 \to r_2}{\to} \begin{bmatrix} 2 & 3 \\ 0 & \frac{1}{2} \end{bmatrix}$$

So we know that it is invertible, and we must go on, all the way. Continuing

$$\begin{bmatrix} 2 & 3 \\ 0 & \frac{1}{2} \end{bmatrix} \overset{r_1 - 6r_2 \to r_1}{\to} \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \overset{\frac{1}{2}r_1 \to r_1, 2r_2 \to r_2}{\to} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad .$$

Now we have to **mimick** the same sequence operations starting with the **identity matrix**, I_2:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \overset{r_2 - \frac{3}{2}r_1 \to r_2}{\to} \begin{bmatrix} 1 & 0 \\ -\frac{3}{2} & 1 \end{bmatrix} \overset{r_1 - 6r_2 \to r_1}{\to} \begin{bmatrix} 10 & -6 \\ -\frac{3}{2} & 1 \end{bmatrix} \overset{\frac{1}{2}r_1 \to r_1, 2r_2 \to r_2}{\to} \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \quad .$$

This is the **ans.**.

Ans. to 2: The matrix is invertible, and its inverse is:

$$\begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \quad .$$

3. True or False (Explain when appropriate)

(a) (1 pt.) If a square matrix has a column consisting of all zeros, then it must be invertible.

Sol. of 3(a): **False**. If it has a column of all zeroes, then it is never invertible (For an $n \times n$, the rank is less than n, so its reduced row echelon form will never be I_n).

(b) (1 pt.) The pivot columns of a matrix are sometimes linearly dependent.

Sol. of 3(b): **False**. The pivot columns of the reduced-row-echelon-form are (different) standard vectors, and obviously linearly independent. By the **column-correspondence property** the same applies to the pivot columns of the original matrix.