Solutions to the Attendance Quiz for Lecture 9

1. For the following matrix, A, determine (a) the reduced row echelon form R of A (b) an invertible matrix P such that $PA = R$.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 2 & 5 \end{bmatrix}$$

Sol.: We append to A the identity matrix I_3.

$$A = \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 3 & 0 & 1 & 0 \\ 2 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

First transform A into **reduced row-echelon from**. To start with, we do

$$(i)\ r_2 - r_1 \to r_2 \quad , \quad (ii)\ r_3 - 2r_1 \to r_3$$

and get

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \end{array}\right]$$

We next do:

$$(iii)\ r_3 - r_2 \to r_3 \quad ,$$

and get

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{array}\right] \quad ,$$

(now it is in row-echelon form). Finally we do

$$(iv)\ r_1 - 2r_2 \to r_1 \quad ,$$

to get

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 3 & -2 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{array}\right] \quad .$$

Now it is in **reduced-row-echelon form**

Final Ans. :

$$R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad P = \begin{bmatrix} 3 & -2 & 0 \\ -1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \quad .$$

You are welcome to check, by doing the matrix-multiplication, that indeed $PA = R$.