Solutions to the Attendance Quiz for Lecture 6

1. Determine whether the given set is linearly independent

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} , \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} , \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\} .$$

Sol. to 1: Form the matrix whose columns are the given vectors, and find its **rank** by converting it to **row-echelon form**. The matrix is:

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} .$$

The first pivot is the first entry at the first row. We want to make the entries below it 0. So we do $r_2 + r_1 \to r_2$ and get

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} .$$

Luckily, the first entry of the third row is already 0. Now the pivot is the $(2, 2)$ entry, and to make the enry below it 0 we do $r_3 + 2r_2 \to r_3$ getting:

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 7 \end{bmatrix} .$$

Now it is in **row-echelon form** and that's enough (no need to go all the way to reduced row-echelon form). There are three pivot entries, so the rank is 3, same as the number of column vectors (or equivalently, the nullity is 0), and this means that this set is **linearly independent**.

2. Determine, if possible, a value of r for which the given set is linearly dependent.

$$\left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} , \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} , \begin{bmatrix} -1 \\ 1 \\ r \end{bmatrix} \right\} .$$

Sol. of 2:

We pretend that r is just a number. Combining the three vectors into a matrix we get

$$\begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & -3 & r \end{bmatrix}$$

It is most convenient to bring the third row to the top and the first row to the second and the second row to the third (it is always OK to permute the rows)

$$\begin{bmatrix} 1 & -3 & r \\ -2 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad .$$

We now make the entries below the $(1,1)$ entry 0. The $(1,3)$ entry is already 0 so we do $r_2+2r_1 \rightarrow r_2$ getting

$$\begin{bmatrix} 1 & -3 & r \\ 0 & -5 & 2r-1 \\ 0 & 0 & 1 \end{bmatrix} \quad .$$

This is already in row-echelon form! There are **always** three **pivot positions** (of course non-zero, pivots are always non-zero!), regardless of the value of r, so there is no choice of r to make some of the pivots disappear (i.e. become zero). This means that it **not possible** to pick any value of r that would make the above three vectors linearly dependent. They are always destined to be linearly **independent** regardless of the choice of r.

Ans. of 2: Impossible.