Solutions to the Attendance Quiz for Lecture 5

1. Determine the value of r for which

$$\mathbf{v} = \begin{bmatrix} 1 \\ r \\ 2 \end{bmatrix} \quad .$$

is in the span of

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad , \quad \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} \right\} \quad .$$

Sol. to 1.: In order for $\mathbf{v}$ to be in the span of the above set, the system of linear equations $A\mathbf{x} = \mathbf{v}$ should have a solution (i.e. be **consistent**) where A is the matrix whose **columns** are the given vector, namely

$$\begin{bmatrix} 1 & -1 \\ 2 & -2 \\ -1 & 2 \end{bmatrix} \quad .$$

The **augmented matrix** of the system is

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & r \\ -1 & 2 & 2 \end{bmatrix} \quad .$$

We now use Gaussian elimination to get it to **row-echelon form** (it is not necessary to go all the way to reduced-row echelon form).

Doing the row operations $r_2 - 2r_1 \to r_2$, $r_3 + r_1 \to r_3$ yields the equivalent system:

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & r-2 \\ 0 & 1 & 3 \end{bmatrix} \quad .$$

We now need to swap the second and third rows, i.e. to peform the row operation $r_2 \leftrightarrow r_3$, getting

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & r-2 \end{bmatrix} \quad .$$

If $r - 2$ is **not** zero than the system is **inconsistent**, so the only way that the system could be **consistent** is for $r - 2$ to be 0 (so you won't get a line $0 \dots 0$ followed by a non-zero number). But $r - 2 = 0$ means $r = 2$.

Ans.: $r = 2$.

Comments 1. About %60 of the students got it right. Some people also found a solution, (i.e. how to express $\mathbf{v}$ with $r = 2$ as a linear combination of the two given vectors. This is **not necessary**. You should read the question carefully and answer exactly what has been asked, not less, but especially **not more**.

2. Another, perfectly correct way is to ask whether $\mathbf{v}$ is a linear combination of the two vectors.

$$\begin{bmatrix} 1 \\ r \\ 2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \quad ,$$

get three equations for the two unknowns c_1, c_2. Solving the first and third equation would give you the values of c_1, c_2 and plugging-in into the second equation would give you the value of r.

2. Find a subset of the following set $\mathcal{S}$ of vectors in R^3 with the same span as $\mathcal{S}$ that is as small as possible.

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \quad , \quad \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix} \quad , \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\} \quad .$$

Sol. of 2: This is a piece of cake! You can immediately kick

$$\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad ,$$

from **any** generating set! (zero is not called zero for nothing!, it is superflous). Now we have left:

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \quad , \quad \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix} \right\} \quad .$$

None of the two remaining vectors can be kicked-out since they are not multiples of each other, so this is the **answer**.

Comments: About %90 of the people got it right.