Solutions to the Attendance Quiz for Lecture 3

1. Perform the indicated elementary operation on

$$A = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 5 & 0 \\ 3 & -1 & 2 \\ 4 & 11 & -2 \end{bmatrix}$$

(i) Interchange rows 2 and 4 .

Sol. to 1(i):

$$A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 11 & -2 \\ 3 & -1 & 2 \\ -4 & 5 & 0 \end{bmatrix}$$

(ii) Multiply row 3 by -3 .

Sol. to 1(ii):

$$\begin{bmatrix} 2 & -3 & 1 \\ -4 & 5 & 0 \\ -9 & 3 & -6 \\ 4 & 11 & -2 \end{bmatrix}$$

Comments: Everyone got it right!

2. The *reduced row echelon form* of a certain system of linear equations is:

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad .$$

Determine whether this system is consistent, and if so, find its general solution. In addition, write the solution in *vector form*.

Sol. to 2: In everyday notation (using as variable names x_1, x_2, x_3), we have:

$$x_1 - 2x_2 = 4 \quad , \quad x_3 = 3 \quad , \quad 0 = 0 \quad .$$

x_1, x_3 are the **basic variables**, and x_2 is the **free variable**. Expressing the basic variables in terms of the free variable(s) (in this example, we only have one free variable), we have:

$$x_1 = 4 + 2x_2 \quad , \quad x_2 = x_2 \, (free) \quad , \quad x_3 = 3 \quad .$$

So this system is **consistent** and this is the **general solution**. To get it in **vector form**, we have:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 + 2x_2 \\ x_2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \quad .$$

Comments: About 70% of the people got it completely right. Some people didn't give the vector forms. Some people got confused and thoughts that there are four variables. Remember this is an **augmented matrix** and the number of variables is the number of columns **take away one** (the last column corresponds to the right hand side of the system of equations).

3. The *reduced row echelon form* of a certain system of linear equations is:

$$\begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad .$$

Determine whether this system is consistent, and if so, find its general solution. In addition, write the solution in *vector form*.

Sol. of 3: Here we see a row (in this example the second row) with **all zeroes** *except* the rightmost one, (1 in this case). This is saying $0 = 1$, and this is **inconsistent** (nonsense!).

Ans. System is inconsistent since it has a row of all zeroes except for the rightmost entry that is non-zero.

Comment: About 90% of the people got it right.