Solution to the Attendance Quiz for Lecture 2

1. Compute the matrix-vector product

$$\begin{bmatrix} 2 & -3 & 1 \\ -4 & 5 & 0 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad .$$

Sol. of 1: It is the **linear combination** of the three **columns** with the c_i's given by $c_1 = 1, c_2 = 2, c_3 = -3$.

$$1 \cdot \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix} + 2 \cdot \begin{bmatrix} -3 \\ 5 \\ -1 \end{bmatrix} + (-3) \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix} + \begin{bmatrix} -6 \\ 10 \\ -2 \end{bmatrix} + \begin{bmatrix} -3 \\ 0 \\ -6 \end{bmatrix} = \begin{bmatrix} -7 \\ 6 \\ -5 \end{bmatrix} \quad .$$

Comments: About %75 of the people got it completely right. Another %15 did it the right way, but messed up the arithmetics. About %10 of the people did it completely wrong, they got as an answer a 3×3 matrix. Remember:

a matrix times a (column) vector is another (column) vector!.

2. If possible, write the vector

$$\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \quad ,$$

as a linear combination of the vectors in $\mathcal{S}$, where

$$\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} , \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Solution of 2.: Any **linear combination** of the vectors in $\mathcal{S}$ can be written as

$$c_1 \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + c_2 \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad ,$$

for *some* constants c_1 and c_2. It is our task to find such c_1, c_2, if possible, that would make this equal to $\mathbf{u}$. This means

$$\begin{bmatrix} c_1 \\ c_1 \\ 2c_1 \end{bmatrix} + \begin{bmatrix} 0 \\ c_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_1 + c_2 \\ 2c_1 + c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \quad .$$

Setting **corresponding entries** equal to each other, this means that we have the **set of equations**

$$c_1 = 1 \quad , \quad c_1 + c_2 = 3 \quad , \quad 2c_1 + c_2 = 4 \quad .$$

Solving we get $c_1 = 1, c_2 = 2$ (the third equation agrees!).

People who got so far would get **most** of the credit, but the **final answer** is to use these c_1, c_2 to express **u**.

$$\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad .$$

This is the final **answer** .

Comments: About %70 of the people got it completely right. Another %15 got it almost right (except for the last step). Another %5 started the right way, but messed up the algebra. About %10 of the people did it completely wrong, or didn't even try. Please review this!