Solutions Attendance Quiz for Lecture 24 (Review Session)

1. Complete the following sentences

a: A vector $\mathbf{u}$ in R^n is a **linear combination** of the set $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ if ...

Ans. to a: there exist real numbers $c_1, \ldots . c_k$ such that $\mathbf{u} = c_1\mathbf{u_1} + c_2\mathbf{u_2} + \ldots + c_k\mathbf{u_k}$.

b: A set of vectors $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ is **linearly independent** if ...

Ans. to b: the only choice of real numbers $c_1, \ldots . c_k$ for which $\mathbf{0} = c_1\mathbf{u_1} + c_2\mathbf{u_2} + \ldots + c_k\mathbf{u_k}$, is the obvious choice of all the c_i being 0, in other words $c_1 = 0, c_2 = 0, \ldots, c_k = 0$.

Or, more succinctly: $[\mathbf{u_1} \ldots \mathbf{u_k}]\mathbf{c} = \mathbf{0} \Rightarrow \mathbf{c} = \mathbf{0}$.

c: A set of vectors $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ is a **generating set** for a subspace V of R^n if ...

Ans. to c: every member of V is a linear combination of the members of $\mathcal{S}$.

d: A set of vectors $\mathcal{S} = \{\mathbf{u_1} \quad , \quad \ldots \quad , \quad \mathbf{u_k}\}$ is a **basis** for a subspace V of R^n if ...

Ans. to d: $\mathcal{S}$ is **both** a generating set and a linearly independent set.

e: An **eigenvalue** of a square $(n \times n)$ matrix A, is a number t such that ...

Ans. to e: there exists a **non-zero** vector $\mathbf{x} \in R^n$ such that $A\mathbf{x} = t\mathbf{x}$.

Or:

if there exists a **non-zero** vector $\mathbf{x} \in R^n$ such that $(A - t\,I_n)\mathbf{x} = 0$.

Or:

If t is a root of the **characteristic equation** $\det(A - t\,I_n) = 0$.

f: An **eigenvector** of a square $(n \times n)$ matrix A is a vector $\mathbf{x}$ in R^n such that ...

Ans. to f: $\mathbf{x} \neq \mathbf{0}$ and $A\mathbf{x}$ is a scalar multiple of $\mathbf{x}$, i.e. there exists a real number t such that $A\mathbf{x} = t\mathbf{x}$ (BTW: t is called the *eigenvalue corresponding to the eigenvector* $\mathbf{x}$).

g: A **pivot entry** in the row-echelon (or reduced-row-echelon) form of matrix is an entry that is ...

Ans. to g: the leftmost **non-zero** entry in its row. (BTW, by construction, it is also the bottommost non-zero entry in its column, i.e. all the entries below it must be 0).

h: An **elementary row operation** is one of the following operations involving either one or two rows of a matrix: ...

Ans. to h:

- $kr_i \to r_i$ (In English: replace the i-th row by k times itself) .
- $r_i \leftrightarrow r_j$ (In English: Interchange row i and row j) .
- $kr_j + r_i \leftrightarrow r_i$ (In English: replace the i-th row by itself plus k times row j) .