Solutions to the Attendance Quiz for Lecture 20

1. Consider the vectors $\mathbf{u}$ and $\mathbf{v}$:

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} -11 \\ 4 \\ 1 \end{bmatrix}$$

(a) Prove that $\mathbf{u}$ and $\mathbf{v}$ are orthogonal to each other.

Sol. of 1a):

$$\mathbf{u} \cdot \mathbf{v} = (1)(-11) + (2)(4) + (3)(1) = -11 + 8 + 3 = 0.$$

Since the dot-product of $\mathbf{u}$ and $\mathbf{v}$ is 0, it follows that they are orthogonal.

(b) Compute the quantities $||\mathbf{u}||^2$, $||\mathbf{v}||^2$ and $||\mathbf{u}+\mathbf{v}||^2$. Use your results to illustrate the Pythagorean theorem.

Sol. of 1b): First, let's compute $\mathbf{u} + \mathbf{v}$:

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -11 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 6 \\ 4 \end{bmatrix} \quad .$$

Now:

$$||\mathbf{u}||^2 = (1)^2 + (2)^2 + (3)^2 = 1 + 4 + 9 = 14$$
$$||\mathbf{v}||^2 = (-11)^2 + (4)^2 + (1)^2 = 121 + 16 + 1 = 138$$
$$||\mathbf{u} + \mathbf{v}||^2 = (-10)^2 + (6)^2 + (4)^2 = 100 + 36 + 16 = 152 \quad .$$

Since $14 + 138 = 152$, Pythagoras was proven right! (at least in this case).

2. Suppose that $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are vectors in R^n such that $\mathbf{u}\cdot\mathbf{v} = 2$, $\mathbf{u}\cdot\mathbf{w} = 3$, and $\mathbf{v}\cdot\mathbf{w} = -2$. Compute $(\mathbf{u} + \mathbf{w}) \cdot \mathbf{v}$.

Sol. of 2: Using the *distributive property* of the dot-product, we have

$$(\mathbf{u} + \mathbf{w}) \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{v} + \mathbf{w} \cdot \mathbf{v}$$

By the *commutative property* we have that $\mathbf{w} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{w}$, so we have

$$(\mathbf{u} + \mathbf{w}) \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{v} + \mathbf{w} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{w} \quad .$$

Now use the **data** of the problem to get that this equals:

$$2 + (-2) = 0 \quad .$$

Ans. to 2: 0.