Solutions to the Attendance Quiz for Lecture 13

1. Evaluate the determinant of the matrix using elementary row operations.

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 3 & 4 & 8 \end{bmatrix}$$

Sol. to 1: We do the first phase of Gaussian elimination, getting it to row-echelon form.

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 3 & 4 & 8 \end{bmatrix} \overset{r_2 - r_1 \to r_2, r_3 - 3r_1 \to r_3}{\to} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & -2 & 5 \end{bmatrix} \overset{r_3 - 2r_2 \to r_3}{\to} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \quad .$$

Now it is an **upper-triangular matrix**, and the determinant is simply the product of the diagonal entries, so it equals $(1)(-1)(3) = -3$.

Ans. to 1: -3.

2. Find the value of c for which the matrix is not invertible

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 4 \\ 2 & 1 & c \end{bmatrix}$$

Sol. of 2: We first do the same thing, pretending that c is a number, and compute the determinant.

$$\begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & 4 \\ 2 & 1 & c \end{bmatrix} \overset{r_2 + r_1 \to r_2, r_3 - 2r_1 \to r_3}{\to} \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 6 \\ 0 & 3 & c-4 \end{bmatrix} \overset{r_3 + 3r_2 \to r_3}{\to} \begin{bmatrix} 1 & -1 & 2 \\ 0 & -1 & 6 \\ 0 & 0 & c+14 \end{bmatrix}$$

So the determinant equals $(1)(-1)(c+14) = -c - 14$. The matrix is not invertible when that determinant happens to be zero, so we set this equal to 0 and solve for c:

$$-c - 14 = 0 \quad .$$

The solution is $c = -14$.

Ans. to 2: $c = -14$.